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Quasi-periodic State and Transition to Turbulence in a Rotating Couette System

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ABSTRACT

Experimental study on flow transition in a rotating Couette system was made by investigating a spatio-temporal velocity field by ultrasonic Doppler method (UVP). Our Taylor-Couette system has a radius ratio $\eta = R_i/R_o = 0.904$ (R_i is the radius of the inner cylinder, 94.0 mm, and R_o that of the outer cylinder, 104.0 mm) and aspect ratio $\Gamma=L/d=20$ ($d=R_o-R_i$, L is the fluid height). Only the inner cylinder is rotated. The Reynolds number R is defined as $R = \Omega R_i d/\nu$ (Ω is the frequency of rotation of the inner cylinder, ν is the kinematic viscosity), and the reduced Reynolds number as $R^*=R/R_c$. In the present configuration, the critical Reynolds number R_c for the onset of TVF is 134.57. The liquid used in the experiments was a mixture of water and 30% Glycerol. The ultrasonic transducer was set outside of one of the stationary end walls, being perpendicular to it with its center on the outer cylinder wall position. The diameter of the ultrasonic beam was 5 mm. Thus the measuring volume of one point is a half disc shape of radius 2.5 mm and of thickness 0.75 mm. The measurement of the velocity profile was focused on the spatial range from 40 mm to 135 mm (i.e., from Z=4d to 13.5d) from the end wall in order to eliminate the end wall effect. This setup of the UVP required a measuring time of 130 msec for a velocity level of a few mm/sec. The 1024 successive profiles were recorded. We performed the measurement for R^* ranging from approximately 10 to >40 for most runs. This covers a flow regime of modulated wavy flow (MWV), for which the onset is at $R^*\approx 9$.

We performed one-dimensional Fourier analysis of the data: power spectrum [1] and energy spectral density (ESD) [2]. These spectra were computed by fast Fourier transformation (FFT) in the time and space domain independently, and obtained space-dependent power spectra and time-dependent energy spectral density for all the data sets. It was found that there are three intrinsic wave modes for the MWV regime and they can coexist. Furthermore, we also found that a significant change in the flow characteristics occurs at $R^*=21$, where the azimuthal wave mode (WVF mode) disappears: beyond it, the magnitude of power and energy of higher harmonic modes behaves very differently for $R^*>21$ than for $R^*<21$.

For investigating the nature of the quasi-periodic state quantitatively, a time domain Fourier analysis is not sufficient, since various spatial modes contribute to the same peak in the power spectrum. It is therefore necessary to decompose the velocity field with respect to space and time simultaneously. We

used a two-dimensional Fourier transformation in this study, since this flow configuration shows not only good temporal periodicity but also good spatial periodicity.

A two-dimensional FFT was computed on space and time coordinate as :

$$S(f,k) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V_z(z,t) \exp(-ikx) \exp(-ift) dx dt$$

where f is a frequency and k a wavenumber. For the present experimental setup of space and time resolution, we can obtain two-dimensional Fourier spectrum on the plane of f=[0,7.57] (Hz) and k=[0,1.33] (mm^{-1}) with resolutions of [14.8mHz, 0.010mm^{-1}].

An example of the results is given by the surface plot in Fig.1. Reflecting the highly spatio-temporal nature of the flow field, the resulting Fourier spectrum has many isolated peaks. Each peak corresponds to each wave mode or their higher harmonics in a sense of two dimension. By filtering out peaks of higher harmonics and their linear combinations and then reconstructing the velocity field, we showed that the decomposition is successful and valid for investigating a flow field[3]. Using such decomposition analysis, we found that the so-called broad band component, which is attributed to chaos, corresponds to a flow motion which moves from roll to roll beyond a roll pair[3].

The same data sets have been analysed using an orthogonal decomposition technique. Among various techniques introduced so far, we adapted the singular system analysis (SSA). The eigenvalue spectrum shows a very sharp decrease for the initial few modes and thereafter it becomes very slow. This reflects the fact that only a few modes contribute to describe the velocity field.

The variation of the first 10 eigenvalues (Fig.2) shows a behavior which is quite similar to the one obtained by 2D-FFT. The first two modes have significant intensity and are approximately constant for $R^*<21$ and then show a sharp decrease at $R^*\approx21$. A broad peak is also seen for $R^*\approx28$.

The total energy occupation (TEO) is defined as the number of modes which occupy 90% of total energy, designated here as N_{90}. It is an index to show the magnitude of the number of participating modes to the flow field.

$$\{\sum_{i=0}^{N_{90}} E_i\} / E_{total} = 0.9$$

where $E_i = A_i^2$ (A_i is a Eigenvalue) and $E_{total} = \sum_{i=0}^{N} E_i$

Fig.3 shows the variation of TEO with respect to Reynolds number. The value of N_{90} increases slowly from 2 to 7 for $9<R^*<21$. Then at $R^*=21$ there is a jump to $N_{90}=35$, it shows a maximum at $R^*=22$ and then slightly decreases. For R^* less than 21, the eigenfunctions of participating modes are more or less harmonic or at least highly periodic, while they are not for the larger R^*. It reaches a local maximum at $R^*\approx22$ and a local minimum at $R^*\approx28$. Since a dominant mode disappears at $R^*=22$, the larger number of modes is needed to occupy total energy, whereas another mode (new mode) appears for $23<R^*<32$ which has a fairly large contribution to the total energy and thus N_{90} becomes smaller. Beyond this point, it shows a gradual increase up to 55 at $R^*\approx100$. This large jump in N_{90} at $R^*=21$ indicates the transition from quasi-periodic flow to turbulence, and at the same time, the transition itself is quite sharp. However, The value of N_{90} itself is still finite and not high, say in the order of 40 to 60.

A global entropy was introduced by Aubry et al.[4] in order to study the order of magnitude of participation of various modes. It shows maximal (equal to 1 when normalised) when energy is uniformly distributed among participating modes. When only a single mode is excited, the entropy is zero.

Fig.4 shows the variation of global entropy with respect to R^*. It is low, around 0.12, for Reynolds numbers less than 19. At around $R^*=21$, it shows a fairly sharp peak which then decreases to 0.13 at around R*=29, and then increases again to about 0.4. This behavior corresponds to the transition scheme discussed earlier: while only two waves exist for MWV, it stays low. When the intensity of these two modes decreases sharply, the entropy increases sharply. When the third wave mode appears above R*≈23, it again decreases, and it increases again when this mode disappears. Such variation reflects the idea of the global entropy that it increases when the number of participating modes increases, among which energy is distributed with equal share. In this context, the behavior is quite similar to TEO as shown in Fig.3. In the present case, a dip of the entropy at around R*≈23 corresponds to the appearance of a new mode which occupies a considerable fraction of the total energy, and it is reflected also as the dip of the curve in N_{90} vs. R*. It shows a considerable fluctuation beyond R*>40, while N_{90} curve in Fig.3 is fairly smooth. The reason for this fluctuation is as yet unclear.

In the range 40<R*<80, the values of N_{90} and the global entropy stay fairly constant. However, N_{90} is around 40-60 being finite and the global entropy H=0.6. From these results, we consider the flow for this range is "soft turbulence"; there is no coherent structure which characterises the flow field, and the energy is shared among the considerable number of modes, but the number of participating modes is still small.

The "soft turbulence" was introduced by Heslot and Casting[20] from the convection experiment in Helium gas. They investigated a scaling law of Nusselt number to Reynolds number and showed a transition scheme from laminar to turbulence through various intermediate states as : convection - oscillation - chaos - transition - soft turbulence - hard turbulence. Our transition scheme is quite similar to this even though our flow configuration is quite different. The correspondence between the two cases is given in Table 1.

Table 1 Similarity in transition states and corresponding flow regime for convection and Taylor-Couette system

Rayleigh Benard	Taylor Couette system	Characteristics
Convection	TVF	Spatially periodic or symmetric
Oscillation	WVF	Temporally oscillating
Chaos	MWV	quasi periodic
Transition	Fast azimuthal wave mode	still coherent-structural
Soft turbulence	Soft turbulence	non-structural with soft spectrum
Hard turbulence	Hard turbulence	non-structural with hard spectrum

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[2] Takeda, Y. Fischer, W.E. and Sakakibara, J., Measurement of energy spectral density of a flow in a rotating Couette system, Phys. Rev. Lett., Vol.70 (1993), p3569.
[3] Takeda, Y., Fischer, W.E. and Sakakibara, J., Decomposition of the modulated waves in a rotating Couette system, Science, Vol.263 (1994), p502
[4] Heslot, F., Casting, B. and Libchaber, A., Transition to turbulence in helium gas. Phys. Rev. A, 36 (1987) 5870;
Casting, B. et al., Scaling of hard thermal turbulence in Rayleigh-Benard convection, J. Fluid. Mech. 204 (1989) 1

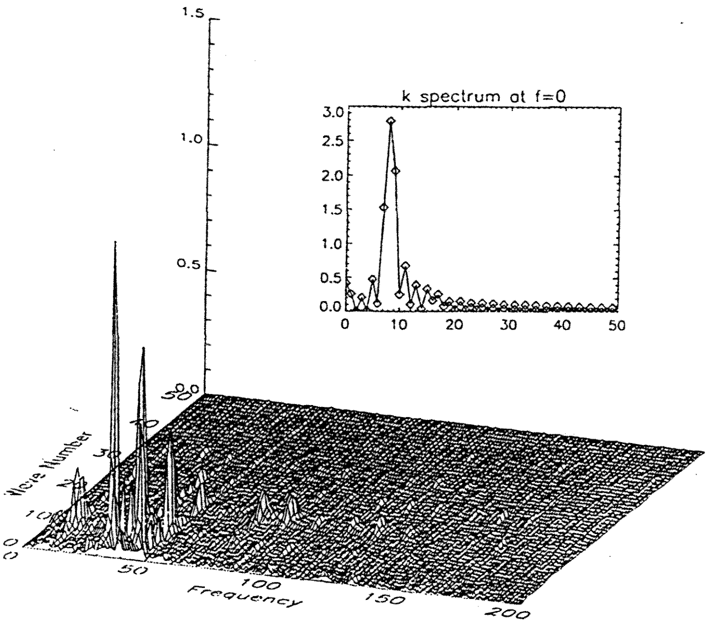

Fig.1 A surface plot of a part of two dimensional Fourier spectrum for the same data as for Fig.3. An insert is a k-spectrum at $f=0$ for clearer view.

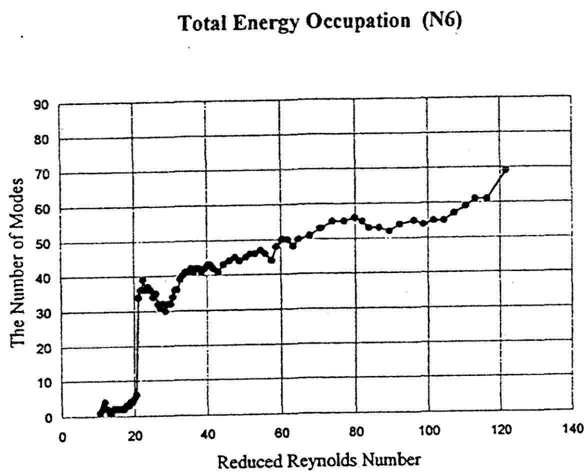

Fig.3 A variation of Total Energy Occupation (TEO) vs R*.

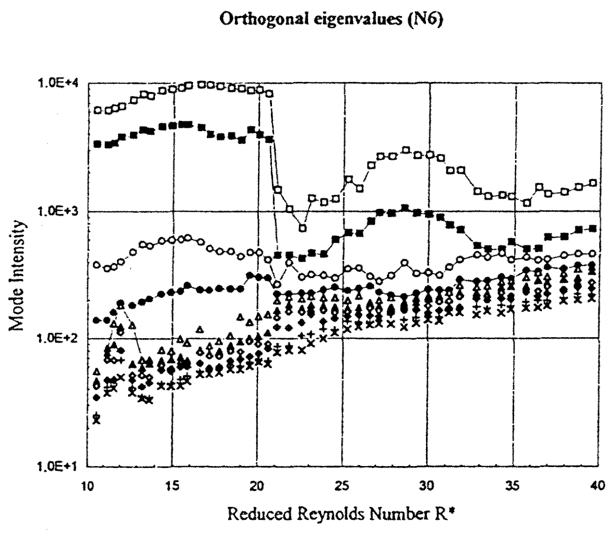

Fig.2 A variation of eigenvalues against R* for the first 10 eigenmodes.

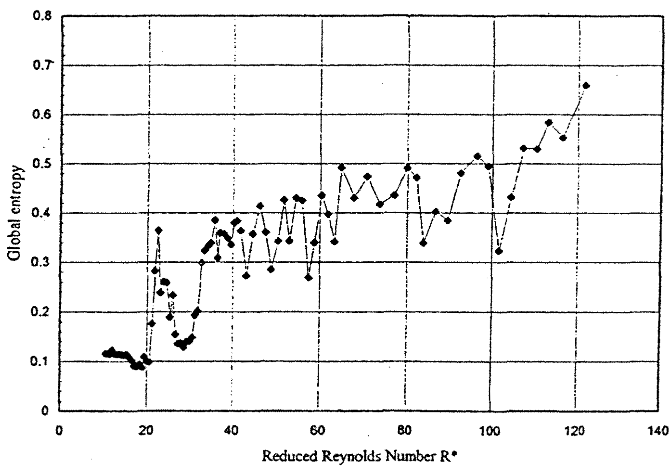

Fig.4 A variation of global entropy vs R*.