Surface roughness determination based on velocity profile measurements

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Summary

Velocity profile measurements in a channel of 0.30 m width and 8 m length were made at the Laboratory of Hydraulic Constructions at the EPF-Lausanne with a UVP probe based on Doppler's echography. The aim of this study was to determine the equivalent sand roughness k_s of the bottom. The slope of the channel is variable between 0 and 1 % and the walls are made of transparent PVC.
The boundary layer's theory developed by Prandtl treats flow in the vicinity of a wall. The friction velocity is given by applying a logarithmic regression curve $u = A_1 \ln(z) + A_2$ to the velocity profile. The calculation becomes an iterative process, because the validity field of this relationship also depends on the friction velocity u_*. The results obtained for various types of bottom surface roughness make possible to draw the following conclusions:

- *The velocity fluctuations in the logarithmic part of the velocity profile are significant. This implies strong variations on the calculation of the parameters A_1 and A_2 of the logarithmic regression curve and a significant dispersion of the roughness k_s.*
- *Vertical distance z in Prandtl's law is measured from a line which passes slightly below the roughness peaks. In general $z_0 \equiv -0.2\, k_s$. By measurements with UVP probe the ultrasounds can reflect on surface elements located at different levels.*
- *A solution is to measure many velocity profiles over the entire length of the channel and to determine the average value of k_s.*

1. Introduction

The pressurised tunnel is one of the significant elements of a hydroelectric power plant. It frequently extends on several kilometres between the reservoir and the surge tank marking the departure of the penstock. It is therefore of major importance to minimize the head losses due to friction in such a tunnel. This can only be done if these losses are predictable in relation with the size of the surface roughness elements. The boundary layer theory is a useful base for the determination of surface roughness which can be evaluate from velocity profile measurements.

2. Theoretical aspects

The head loss h_r [m] divided by the length of reach concerned L [m] is called friction slope J_f [-]. The common relation used to express J_f is the Darcy-Weisbach formula (1):

$$J_f = \frac{h_r}{L} = \frac{V^2}{2g} \cdot \frac{f}{D} \qquad (1)$$

D : section diameter	*[m]*
V : mean flow velocity	*[m/s]*
g : gravitational acceleration	*[m/s^2]*
f : friction coefficient	*[-]*

The friction or resistance coefficient f was investigated experimentally by Prandtl [1]. An analytical relation was later proposed by Colebrook and White (2) for the expression of f in turbulent flow condition over random surface roughnesses in opposition to the uniform sand distribution used in the Prandtl experiments [2].

$$\frac{1}{\sqrt{f}} = -2 \cdot \log\left[\frac{k_s}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right] \qquad \text{valid for } Re > 5000 \text{ with} \qquad (2)$$

$$Re = \frac{V \cdot D}{\nu} \qquad (3)$$

k_s: equivalent sand roughness *[m]*
Re : Reynolds number *[-]*
ν: kinematic viscosity *[m²/s]*

k_s corresponds to the sand grains diameter, homogeneous and uniformly distributed, which would cause the same pressure loss as the surface roughness of a trade conduit.

The value of k_s depends on the height, forms, density and distribution of roughness elements.
Moody was the first to propose a graphical representation of the relation of Colebrook and White (Fig. 1). This diagram give the head loss factor f as a function of Reynolds number Re and relative roughness k_s/D. The purpose of the study then becomes to determine the equivalent sand roughness k_s of a surface indifferently.

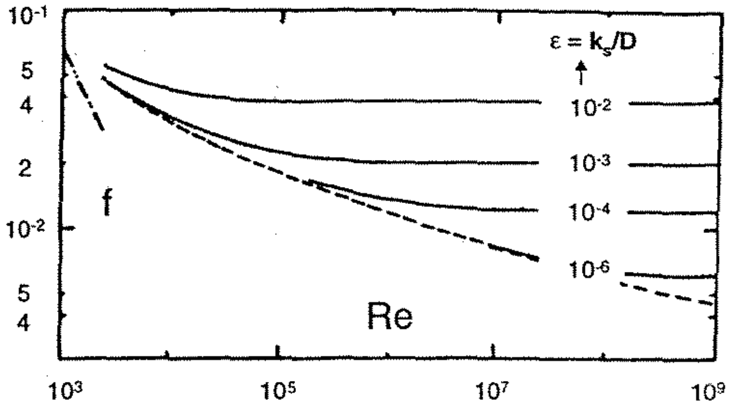


Figure 1 : Diagram of Moody-Stanton

The boundary layer theory, developed by Prandtl, treats the flow in the vicinity of a wall [1]. This flow can be subdivided in two zones:

1. the close to wall, of weak thickness, called boundary layer, where the influence of friction forces is significant;
2. the zone far away from the wall, called free fluid, where the influence of friction forces becomes negligible.

The Prandtl's law in the interior zone of the flow is given by (4).

$$\frac{u}{u_*} = \frac{1}{\kappa}\ln\left(\frac{z}{k_s}\right) + B_r \qquad (4)$$

u_*: friction velocity *[m/s]*
z : outdistance starting from the bottom *[m]*
κ: universal constant of Von Karman *[-]*
B_r: constant of integration *[-]*

The validity field of (4) is defined as $60 \le \frac{z u_*}{\upsilon} \le 500$ which is equivalent with $0.01 \le \frac{z}{\delta} \le 0.2$, δ [m] is the distance from the bottom to which one notes maximum velocity.

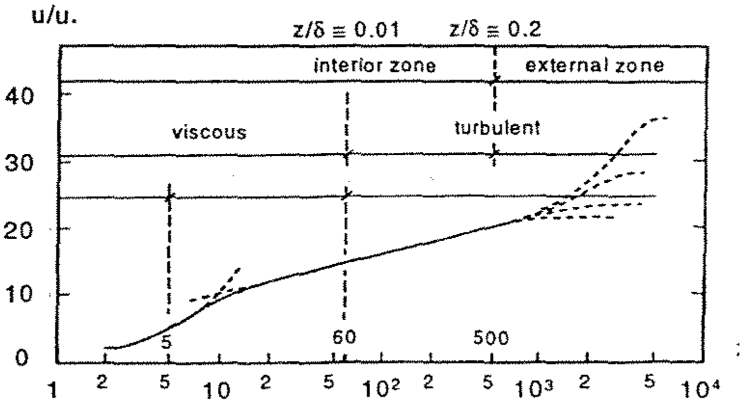
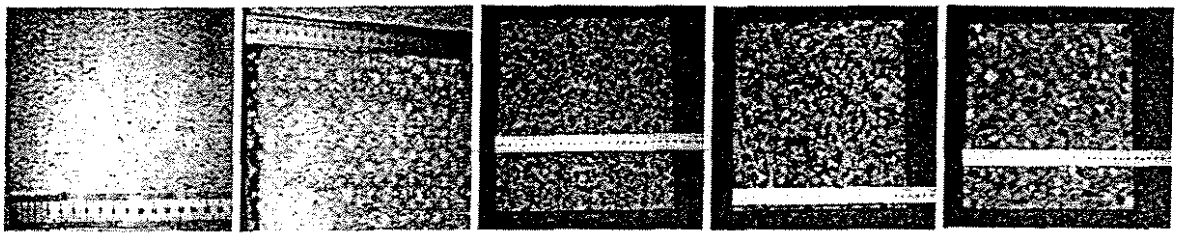


Figure 2: Validity field of the Prandtl's law

3. Experimental study

Velocity profiles measurements with a UVP X-3-PS probe [6] were made in a flume of 30 cm width and 8 m length. The data was collected at a frequency of 10 Hz and a 0.748 mm grid spacing along the axis of the transducer. This axis made an angle of 60° with the channel bottom, corresponding to one measurement every 0.648 mm along the vertical axis. The slope is variable between 0 and 1% adjustable using a crank. The side walls are made of transparent PVC and the tested bottom surfaces were following:

- smooth concrete (Fig. 3a),
- concrete with regularly bored holes representing 4.5 % of the total surface (Fig. 3b),
- Garden flagstones with some different grain sizes and forms (Fig. 3c-3e).



a) b) c) d) e)

Figure 3 : Tested bottom surfaces

In order to obtain a broad channel flow condition, i.e. with negligible influence of the walls, a flow discharge of 6 l/s was used.
The friction velocity u_* and roughness k_s was calculated for each basic type by applying the boundary layer theory described previously. These two steps are described hereafter:

Determination of u_*

1. First choice of the validity interval of (4) : (z_{min}^1 ; z_{max}^1).
2. Application of the corresponding regression curve : $u = A_1^1 \ln(z) + A_2^1$
3. Calculation of the friction velocity : $u_*^1 = \kappa \cdot A_1^1$
4. Checking of the result validity : $\frac{z_{min}^1 \cdot u_*^1}{\upsilon} \geq 60$ and $\frac{z_{max}^1 \cdot u_*^1}{\upsilon} \leq 500$
5. Validity field satisfied : calculation of k_s
   Validity field not satisfied : new choice of the validity interval.

Determination of k_s

Once the friction velocity calculated, it is easy to determine the roughness k_s with (5):

$$k_s = \exp\left[\left(\frac{A_2}{u_*} - B_r\right) \cdot (-\kappa)\right] \tag{5}$$

The B_r value in (5) is 8.5, corresponds to a rough flow condition. This is checked using the relation $\frac{u_* \cdot k_s}{\upsilon} \geq 70$ proposed by Nikuradse [3]. If not the case, the flow condition can be smooth or in transition state and the value of B_r becomes a function of $u_* k_s/v$. Two formula were proposed by Nikuradse and Krishnappan for this case [5]. The resolution must be made in a graphic way considering that an analytical solution doesn't exist.

4. Results

An example of a measured velocity profile is given on fig. 4. It is noted that the fluctuations speed in the logarithmic curve part of the profile are significant, which makes difficult a precise estimation of the friction speed u_* and roughness k_s. Figure 5 shows the Prandtl's law adjusted to the logarithmic part of the velocity profile, in the interval defined on figure 2. From the adjusted coefficients A1 = 0.0848 and A2 = 0.82, the friction velocity and the roughness values can successively be calculated.

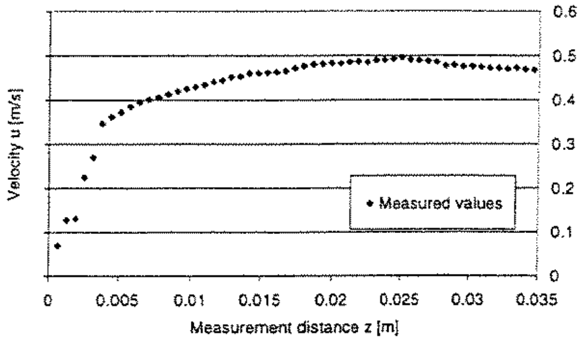


Figure 4 : Velocity profile measurement.

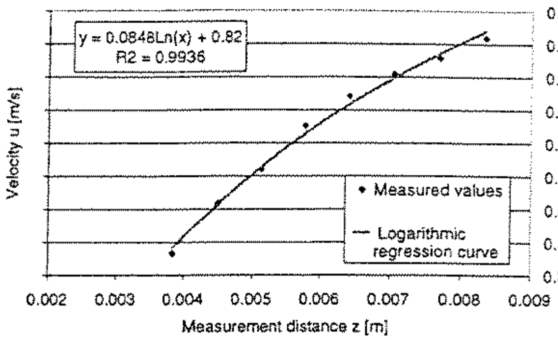


Figure 5 : Logarithmic regression curve in the interior zone.

The results of k_s for each bottom type are given in table 1.

Tested plates		**k_s [mm]**	**Comments**
Smooth concrete (figure 3a)		0.37	Good agreement with the values given in literature.
Concrete with 4.5% of holes (figure 3b)		0.59	Weak roughness increase compared to the smoothes case.
Garden flagstone VERONA (figure 3c)	(A)*	15.23	For equivalent projected surfaces, roughness k_s strongly depends on the shape of the grains. Kibbled grains, with square edged boards, produce a higher value of k_s than grains with round form.
Garden flagstone RIVIERA (figure 3d)	(B)*	1.67	
Garden flagstone JURA (figure 3e)	(A)+(B)	5.66	

(A) : Kibbled grains ; (B) : Rounded grains.

Table 1 : Calculated k_s for different surface types.

5. Conclusions

The results obtained in the determination of the sand equivalent roughness for various types of surfaces make it possible to draw the following conclusions:

- The velocity fluctuations in the logarithmic part of the velocity profile are significant [5]. This implies strong variations on the calculation of the parameters A_1 and A_2 of the logarithmic regression curve and a significant dispersion of the surface roughness k_s.
- Vertical distance z in Prandtl's law is measured from a line which passes slightly below the roughness peaks. In general $z_0 \cong -0.2\ k_s$ [3]. By measurements with UVP probe the ultrasounds can touch parts of surface which are on different levels.

The proposed solution is to make velocity profile measurements over the entire length of the channel and to seek the average value of k_s.

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