Development of Rheometry based on UVP for visco-elastic liquid

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As a method to estimate the property of visco-elastic liquid, UVP is utilized for unsteady flow field which arises during spin-up and spin-down of container. This flow configuration provides necessary information to determine the elasticity and the viscosity of the target fluid. First, a numerical simulation is performed to understand how the velocity profiles along UVP-measurement line is obtained. Secondly, the unsteady flow of PAA solution that is a typical example of visco-elastic fluid is actually measured with UVP. Since the property of PAA solution is characterized by three factors; viscosity, elasticity, and yield stress, these factors are estimated with Maxwell and Bingham models using least square approach. In our study, the least square approach is applied to the spatio-temporal two-dimensional distribution of velocity. The results are validly obtained with this approach, and thus we conclude that the present flow configuration is feasible as a UVP-based rheometry.

Keywords: Rheological properties, Transient shear response, Visco-elasticity, Yield stress

1 INTRODUCTION

Rheology for liquid foods are positioned in highly important research field in a part of life science and food process engineering. They often show very different behavior from ordinary Newtonian fluids. The visco-elastic properties of liquid foods sometimes cause accident in swallowing them, and also relate to quality control of food manufacturing process. In order to estimate these properties in visco-elastic fluids, a high accuracy but convenient method is desired to be built up. Conventional methods for estimating them utilize indirect information obtained from differential pressure in a tube or torque caused by the wall shear stress. In those methodologies, the internal flow structure which will depend sensitively on the visco-elastic properties is consequently kept invisible so that the local flow information cannot be assessed. This problem slows down the scientific understanding of the rheological behavior.

Since ultrasonic velocity profiler (UVP)[1] can acquire the velocity profiles as a function of time, it has a potential to become a high-performance rheometry. Because of ultrasonic principle, most of liquid foods can be measured regardless the opaqueness, such as milk, yogurt, and chocolates. In this study, our objective is set to develop such a rheometer based on UVP, by choosing the best optimized flow configuration. The best optimizing means that the spatio-temporal two-dimensional velocity information obtained by UVP would be fully utilized to accurately detect the material properties of the target fluid. To this end, the transient shear response of fluid driven by sudden spinning in a container is selected.

In this paper, we describe about the method of UVP-based rheometry, numerical analysis of the visco-elastic fluid in the spinning flow, and the examination using polyacrylamide (PAA) solution as the demonstration. In the final part of the paper, the algorithm for obtaining the visco-elastic properties from UVP data is proposed and its validity is discussed.

2 FLOW CONFIGURATION

Figure 1 shows general description of the present flow configuration. There are advantages with this flow configuration in comparison to pipe flows that are also utilized as UVP-base rheometry[2-6]. The key point in spinning flow is that it enables to acquire the transient shear response from quiescent state. Therefore, wall-clinging effect for the elasticity, fluidized behavior departing from the yield point, and shear-thinning effect are all captured simultaneously with a single run of the measurement. Furthermore, the steady state converges to rigid rotation for any type of fluids, we need not to consider the dependence on initial conditions, which often takes place in the case of pipe flow. This is also a important merit for guaranteeing the reproducibility of the measurement. Thus, a small-volume sampling of target fluid pouring into the rotor provides full information of the theological properties when the spinning flow is utilized.

As shown in Fig. 1, the measurement line is set at the distance of 7mm from center line of a cylindrical container. The container is made of acrylic resin, and has 150mm in diameter and 2mm in thickness. We checked the echo caused on the sidewall of the cylinder to define the range of measurement. As shown in Fig. 2, the temperature of the liquid is controlled to be constant at 20deg C, and an ultrasonic absorber is put on the backside of the transducer. The basic frequency is set at 4MHz. As ultrasonic reflector in fluid, high-porous polymer particles (HP20SS) are mixed. The speed of sound for 20deg C PAA solution is 1500m/s. Sampling period is 20msec, and the number of profiles is 1024. Spatial resolution is 0.75mm while the number of channels is 230. The flow velocity resolution is 1.95mm/s. The rotational speed of the container can vary from 50 to 150rpm.

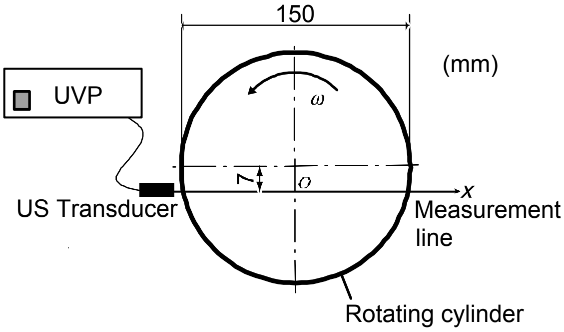


Figure 1: Definition of measurement line in container

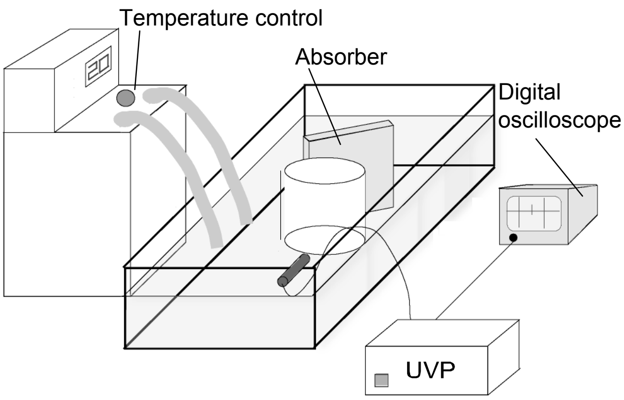


Figure 2: Experimental setup for PAA solution

3 NUMERICAL SIMULATION

Before conducting the experiment, how the velocity profiles are obtained on the measurement line must be evaluated. Ordinary visco-elastic liquid has a yield stress at which the flow is released from the elastic potential to be fluidized and stretched. Therefore we employ Bingham model, which is one of non-Newtonian fluid model in the numerical simulation. The simulation is conduced with axisymmetric cylindrical coordinate system, using the following basic equation.

$$\rho \frac{\partial v}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \tau) \tag{1}$$

Here ρ is density of fluid, t is time, and v is tangential (azimuthal) velocity at the radial position, r. Since there is no pressure gradient in the azimuthal direction as far as any wavy behavior does not appear, the pressure term disappears in Eq.(1). We assume that the radial flow be quiescent so that no equation is required for it. This is valid for a long cylinder in the axial direction (however, the experiments show certain flow in this direction as we mention later). As a model of visco-elastic fluid, we use the following Bingham model.

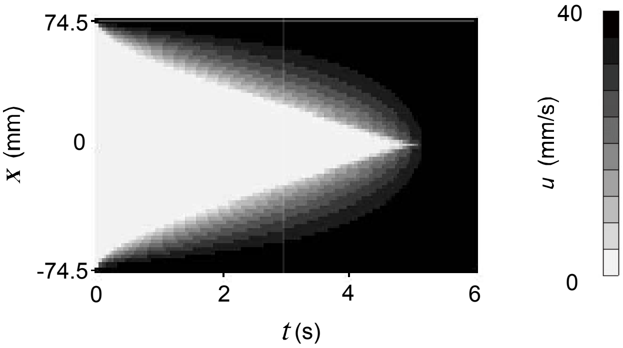
$$\begin{aligned} &\tau = E\gamma = E\int \frac{\partial \gamma}{\partial t}dt = E\int \frac{\partial v}{\partial r}dt && (\tau < \tau_y) \\ &\tau = \mu \frac{\partial v}{\partial r} + \tau_y && (\tau \geq \tau_y) \end{aligned} \tag{2}$$

Here, γ stands for shear strain rate, μ the viscosity, and τ_y the yield stress. As shown in eq. (2), the formula of the shear stress switches to be either of two dependent on the local instantaneous shear stress, τ. In the case of the shear stress smaller than the yield stress, the shear stress is caused by the shear displacement that equals to integrated shear rate respect to time. Beyond the yield stress, the shear stress additionally increases linearly to the shear rate similar to Newtonian fluid.

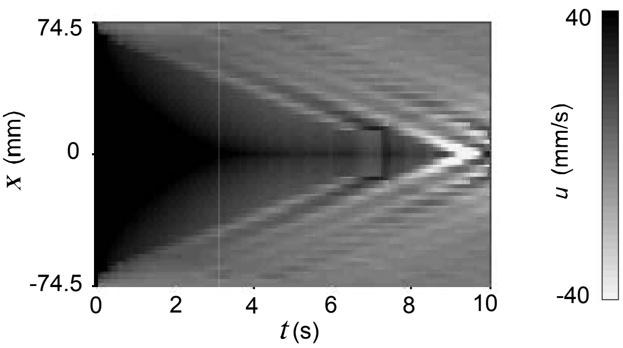
As examples, the following conditions are supposed in the numerical simulation, which approximates the experimental conditions: top rotational speed 50rpm, μ=0.05Pas, E=0.05Pa, τ_y=0.15Pa.

Figure 3 shows the results of the numerical simulations. The grayscale bar indicates the flow velocity component along the measurement line (see Fig. 1). In the case of rigid rotation, this velocity component gets uniform on the line, and thus the transient response of the fluid by the spinning is visualized by the change in the grayscale. As shown in Fig. 3 (a), when the rotation suddenly starts at t=0, the fluid in the cylinder is accelerated near the sidewall of the container. In the beginning of the spin up, the shear stress exceeds the yield stress for the high shear rate near the wall. Therefore, the high-speed region expands in the container similarly to Newtonian fluid. There is also a certain contribution of the elasticity to the acceleration, which exists in the central region of the container having less shear rate. In fact, a small oscillatory flow is overlapped, however it is hidden in monotonic increase of the velocity. We confirm that changing the three parameters μ, E and τ_y extensively, a number of different spin up responses are obtained.

As shown in Fig. 3(b), the influence of the elasticity is more obviously observed in the process of spin down. Here this sample result is obtained under the following conditions; μ=0.15Pas, E=0.10Pa, and τ_y=0.5Pa. After the sudden stop of the cylinder from the steady rotation at t=0, the velocity in wide region of the cylinder changes accompanying significant oscillation in time and in space. The shear stress is caused largely near the wall in the beginning, and its effect quickly propagates into the central region to form a wedge typed dark region in the graph. This is Newtonian effect. After that, the slow-speed region involves velocity fluctuation for a long time. Hence a stripe pattern of negative velocity (reverse flow) regions takes place due to the elastic bouncing of the low shear rate fluid. Furthermore, this bouncing behavior converges in the central region as time elapses, and the local amplitude of the oscillation expands there. Therefore the spin down process is more sensitive to the elasticity when the flow velocity is captured on the measurement line assigned in this study.



(a) Spin-up flow



(b) Spin-down flow

Figure 3: Spatio-temporal two-dimensional velocity distribution of visco-elastic fluid obtained by numerical simulated with Bingham model at 50rpm

4 UVP MEASUREMENT RESULTS

Figure 4 shows a sample of measurement data obtained by UVP for real PAA solution. It shows the spatio-temporal velocity distribution in the same manner as in the last chapter. It is noted that the velocities in the region -74.5mm <*x*< -60mm could not be obtained correctly because of wall interference of the pulsed ultrasound. However, those on the opposite side are validly measured so that it is not a serious lack of the information to determine the rheological properties later.

In general, the measurement result has a good agreement with the numerical simulation in qualitative sense. For instance, we could find reverse flow in the process of spin down for the elasticity of the fluid. The difference from the numerical simulation is that the velocities are obtained asymmetrically for the center of the rotating cylinder. This is due to the flow velocity component induced in the radial direction. In the case of finite cylinder in the axial direction, the spin up causes radial inward flow in the middle part, and the spin down cause radial outward flow there, owing to the difference in centrifugal acceleration. Namely the finite cylinder forms a secondary flow inside during the sudden spinning. Since UVP detects the velocity component sensitively in the radial direction (see Fig. 1), the data are biased strongly to be asymmetric. However the actual radial velocity component is still ignorable relatively to the tangential one so that the flow itself is approximately axisymmetric. Namely the secondary flow that is detected strongly in UVP can be removed in the post-processing to deduce the rheological properties, as mentioned next.

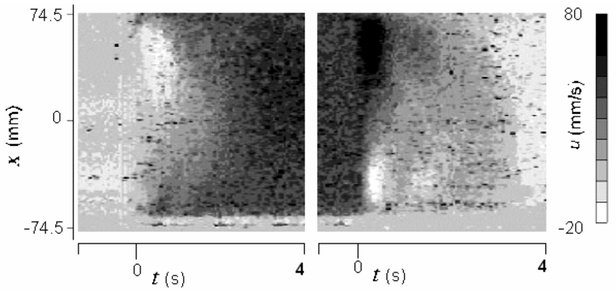


Figure 4: Spatio-temporal velocity distribution of PAA solution, measured by UVP at 50rpm.

5 DATA ANALYSIS AS RHEOMETRY

The radial convection is axisymmetric, and the measurement line of UVP covers the range beyond the center of the cylinder. Thereby the tangential velocity of the flow, v, can be calculated from the single velocity profile of UVP. With this preparation, we can estimate the shear response of the fluid as we explained in the chapter 2.

The data conversion from the raw velocity to the tangential velocity provides another spatio-temporal distribution as shown in Fig. 5. The figure represents the result of a single run that includes both the spin up and the spin down.

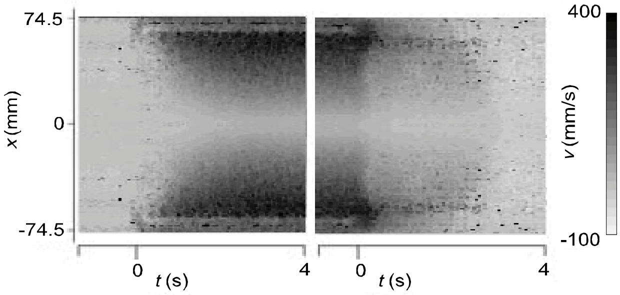


Figure 5: Spatio-temporal distribution of v, tangential velocity component.

The relationship between the local tangential velocity, v, and the local shear stress, τ, is described by the following equation when the flow obeys Maxwell model.

$$\frac{\partial v}{\partial r} = \frac{\tau}{\mu} + \frac{1}{E}\frac{\partial \tau}{\partial t} \tag{3}$$

Coupling this equation with Eq.(1), the tangential velocity must behave with the following equation.

$$\frac{\partial^2 v}{\partial r^2} = \frac{\rho}{E}\frac{\partial^2 v}{\partial t^2} + \frac{\rho}{\mu}\frac{\partial v}{\partial t} \tag{4}$$

In Eq. (4), the elasticity, *E,* and the viscosity, μ, are treated as unknown constants while the tangential velocity, *v*, is given by the UVP measurement as spatio-temporal two-dimensional distribution. Here the density, ρ, is treated as known constant because it is easily measured by measuring the mass and the volume.

In mathematical principle, the elasticity and the viscosity are uniquely determined when Eq. (4) is applied two times at arbitrary instants in the UVP data. For ensuring their accuracy, a lot of information from UVP is better applied. Therefore we propose an algorithm based on least square approach as follows. As the error from Eq. (4), the cost function is defined as follows.

$$G(E,\mu) = \iint \left\{ \frac{\rho}{E}\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 v}{\partial r^2} + \frac{\rho}{\mu}\frac{\partial v}{\partial t} \right\}^2 drdt \tag{5}$$

The value of the cost function *G* varies with *E*, and μ. The best estimates of μ and *E* are obtained when *G* is minimized. This condition can be found out by searching the conditions satisfying $\partial G / \partial \mu = 0$, and $\partial G / \partial E = 0$, simultaneously. Deriving mathematically, these conditions are fulfilled when the viscosity and the elasticity are given by the following equations.

$$\mu = \frac{b^2 + ad}{bc + ae}, \quad E = \frac{ae + bc}{cd - eb} \tag{6}$$

Here, *a*, *b*, *c*, *d* and *e* are given below.

$$a = -2\rho \iint \left(\frac{\partial^2 v}{\partial t^2} \right)^2 drdt \tag{7}$$

$$b = -\rho \iint \frac{\partial v}{\partial t}\frac{\partial^2 v}{\partial t^2} drdt \tag{8}$$

$$c = 2\iint \frac{\partial^2 v}{\partial r^2}\frac{\partial^2 v}{\partial t^2} drdt \tag{9}$$

$$d = 2\rho \iint \left(\frac{\partial v}{\partial t} \right)^2 drdt \tag{10}$$

$$e = \iint \frac{\partial v}{\partial t}\frac{\partial^2 v}{\partial r^2} drdt \tag{11}$$

These five values become zero in the case of quiescent and rigid rotating conditions. Therefore the integral range for *G* in Eq.(5) must include the transient response of velocity. In addition, these five values are quite sensitive to the measurement error since all the value is obtained from temporal and spatial derivatives of the velocity. We have applied some noise reduction filters to the data to overcome this problem.

The estimated properties of the present PAA solution are shown in Table 1. PAA is known as having shear-thinning effect and constant elasticity. The estimated properties are in agreement with them.

Table 1: Properties of 0.24wt% PAA solution assessed by calculation

Rotational Speed	Process	μ(Pas)	*E*(Pa)
50rpm	Spin-up	2.51	1.39
	Spin-down	2.53	1.31
100rpm	Spin-up	2.49	1.47
	Spin-down	2.56	1.24
150rpm	Spin-up	2.38	1.52
	Spin-down	2.49	1.41

6 CONCLUSIONS

A method to estimate the rheological properties of visco-elastic liquid is proposed. It is enabled by use of UVP that is applied for the line measurement in a sudden spinning of fluid in a cylindrical container. As numerical simulation shows, this configuration provides rich information in terms of shear response. Thereby it is suitable for the inverse analysis that estimates the fluid properties from transient velocity data. A least square approach applied for full spatio-temporal velocity distribution realizes to estimate validly the viscosity and the elasticity of a PAA solution.

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