# Velocity profiling of viscoelastic fluids around a falling sphere

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A method to measure two-dimensional velocity vector field around a falling sphere in viscoelastic fluids is developed by means of UVP (Ultrasonic Velocity Profiling). It bases on single-line measurement of fluid velocity component coupled with equation of continuity under the assumption of axisymmetric field. The aim of the study is to validate the UVP measurement performance for the application to opaque viscoelastic fluids. The validation is made with PIV (Particle Image Velocimetry) measurement under the same condition in order to compare the velocity vector field obtained. From the comparison, the flow structure such as toroidal circulation beside the sphere has been validly measured. Instead, negative wake phenomenon so-called has not been extracted by UVP for two reasons in the present technique. The handicap of UVP for viscoelastic fluid and the method of improvement are also discussed.

**Keywords:** Viscoelastic fluid, Falling sphere, Ultrasonic velocity profiling, Multi-dimensional flow

## 1 INTRODUCTION

For decades, several interesting flow behaviors have been reported in terms of falling sphere system with viscoelastic fluids. One of the most noticeable phenomena is reversing flow in a wake of the moving sphere, called “negative wake.” The negative wake is induced by combined stress [1], which is difficult to be characterized directly in most of commercial rheometers. Therefore falling sphere system should be handled by multi-dimensional velocimetry when its rheological response of fluid surrounding the sphere is investigated. However, most of non-Newtonian fluids in application are opaque, and hence optical method such as PIV or LDV measurements are unavailable as long as visible light is applied. X-ray and Neutron scanning are applicable for limited types of fluid while those are generally poor in spatial resolution.

Because of these backgrounds, a method to measure multi-dimensional fluid velocity vector distribution in falling sphere system in non-optical way is targeted in the present study. The method is established by means of ultrasonic velocity profiling (UVP[2]). There are two types of methodology in UVP measurement for acquiring two-dimensional velocity distribution. One is to simply employ two UVP-measurement lines with different angles in the target flow. This set-up enables us to conduct two-component velocity measurement. When flow field is completely steady state on Lagrangian frame that moves with the sphere, the two-dimensional two-component (2D-2C) velocity distribution can be reconstructed. Another technique is to couple the velocity profile along single UVP measurement line with equation of continuity. This combination allows space-time two-dimensional flow measurement for unsteady flows. When the sphere velocity and the flow around the sphere are steady, the space-time frame can be converted to space-space two-dimensional coordinate.

In this paper, we describe the method and the result of the latter case, i.e. single-UVP measurement coupled with equation of continuity, for detecting some key characteristics in viscoelastic fluids that take place around the falling sphere. The measurement performance is validated with the data obtained by particle image velocimetry (PIV) applied for the same experimental conditions. Since some technical problems come out in the result, we also discuss the reasons and the future improvements in the final part of the paper.

## 2 EXPERIMENTAL SET-UP

A schematic illustration of the experimental apparatus is shown in Figure 1. It mainly consists of a cylindrical container, a reservoir and a sphere. The inner diameter of the cylindrical container is 124 mm. It is filled with a test fluid, 1 wt% polyacrylamide (PAA, AP805C Dia-Nitrix) solution. As tracer particles, high-porous polymer particles (HP20SS, Mitsubishi Chemical) are mixed into the solution both for UVP and PIV measurements. The liquid top surface is set at 406 mm height from the bottom of the cylindrical container. The cylindrical container is settled in the reservoir that is filled with water in order to relax reflections of ultrasonic pulse on the container wall, and to prevent refractions of light in PIV measurement. A sphere is hung up with a thread just under the top free surface. The sphere falls along center axis of the cylinder straightly after the thread is cut. The mass of the sphere is 47.8 g, and diameter is 39.5 mm.

The rectangular area shown in Figure 1 indicates the measurement area of PIV. This area is illuminated vertically with a laser sheet. A high-speed video camera (FASTCAM-MAX Photron) is used to photograph the particle images. Frame rate and spatial resolution are 60 fps and 0.17 mm/pixel, respectively.

In UVP measurement, the ultrasonic transducer is located at 186 mm from the top surface of test liquid. The direction of the measurement line crosses perpendicularly with center axis of the cylinder. A UVP monitor (UVP-DUO, MET-FLOW) is used for Doppler velocity computation. Spatial resolution is 0.37 mm while the number of channels is 373. Velocity and time resolutions are 0.15 mm/s, and 0.084 sec, respectively.

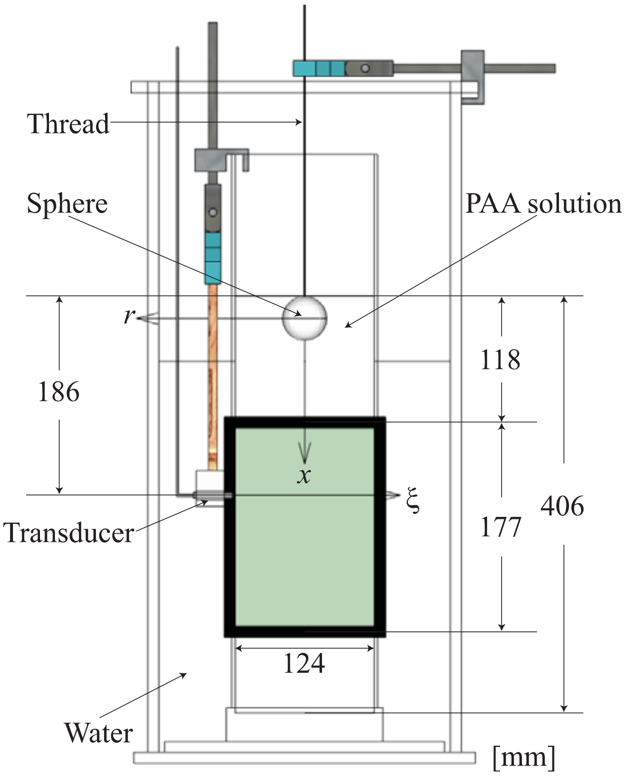

Figure 1: Schematic illustration of experimental apparatus

## 3 FRAME CONVERSION

UVP obtains radial velocity component $u_r$ on the measurement line as a function of $r$ and $t$. When translational velocity of the sphere is given as $U$, temporal information of the velocity can be converted into spatial distribution around the sphere using eq. (1).

$$dx = Udt \tag{1}$$

Here $U$ can be treated as constant when the sphere reaches terminal falling velocity. Since the test liquid is incompressible, equation of continuity stands regardless to rheological properties as

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_r}{\partial r} + \frac{u_r}{r} = 0 \tag{2}$$

where $u_x$ is axial velocity component. From eqs. (1) and (2), axial-radial two-dimensional distribution of liquid velocity is estimated by

$$u_x(x,r) = \int \frac{\partial u_x}{\partial x} dx = -\int \left( \frac{\partial u_r}{\partial r} + \frac{u_r}{r} \right) Udt \tag{3}$$

where initial condition; $u_x = 0$ at $t = 0$, is applied for temporal integration of the kernel function.

It is noted that the spatial resolution of $u_x$ in the axial direction varies with the sphere velocity $U$ and temporal resolution of UVP. In order to keep the homogeneous resolution in 2-D space, a suitable averaging filter is introduced into preprocessing of UVP data. The present case employs median value filter that has 15 ($r$ direction) times 3 ($x$ direction) matrix.

## 4 RESULTS

### 4.1 PIV measurement

The instantaneous velocity vector field around the falling sphere is shown in Figure 2. Each vector field is obtained in different shot in the same movie. The distribution is shown on cylindrical coordinate system, $x$-$r$, whose origin corresponds to the center of the sphere. Both of axes are non-dimensionalized with the radius of the sphere, $R$ = 19.8 mm. Velocities are non-dimensionalized with the terminal velocity of the sphere, $U$ = 62.5 mm/s.

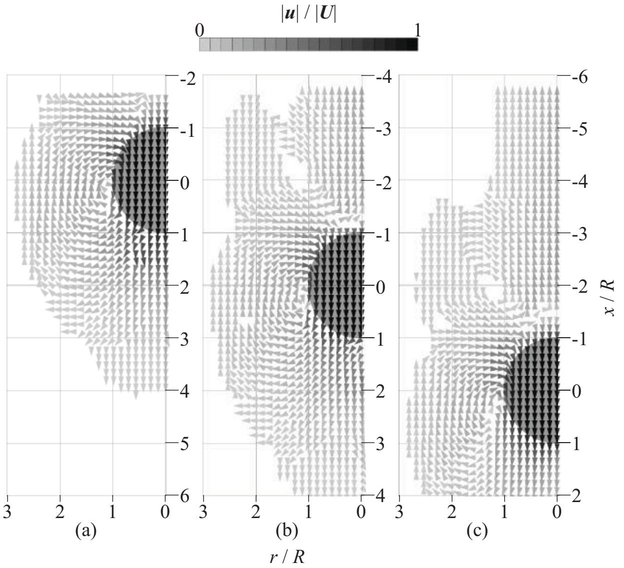

Figure 2: Velocity vector fields obtained with PIV at different shot of sphere

As seen in the results, the negative wake [1] is identified at $x / R < -2$, which has upward velocity for long distance in the downstream direction. The negative wake is considered as strong elastic recovery of the liquid behind the sphere after large strain is subject to the liquid beside the sphere. In addition, two circulation zones are observed in

Figure 2 (b) (c). One is at the side of the sphere ($x$ / $R$ = 0, $r$ / $R$ = 1), and another is at the side of negative wake ($x$ / $R$ = -2, $r$ / $R$ = 1.2). The former circulation is caused by an existence of the wall of the cylindrical container. The latter one is a specific phenomenon for viscoelastic flow, which is connected with the negative wake.

### *4.2 UVP measurement*

The two-dimensional distribution of $u_r$ obtained by UVP is shown in Figure 3 (a). The center of the sphere is estimated from the flow pattern which we have known from PIV measurement. The region around the point ($x$ / $R$ = 1, $r$ / $R$ = 1) indicates that fluid is pushed away by the sphere, i.e. potential displacement. The region around the point ($x$ / $R$ = -2, $r$ / $R$ = 1) has a flow direction approaching the sphere.

Figure 3 (b) shows the velocity vector field obtained by applying the equation of continuity to the data of Figure 3 (a). There is a circulation identifiable around $x$ / $R$ = 0, $r$ / $R$ = 1.5. However, compared to Figure 2 (c), the magnitude of velocity vector near the sphere is much smaller in Figure 3 (b), and a negative wake is not seen clearly. The reason of these differences is discussed below.

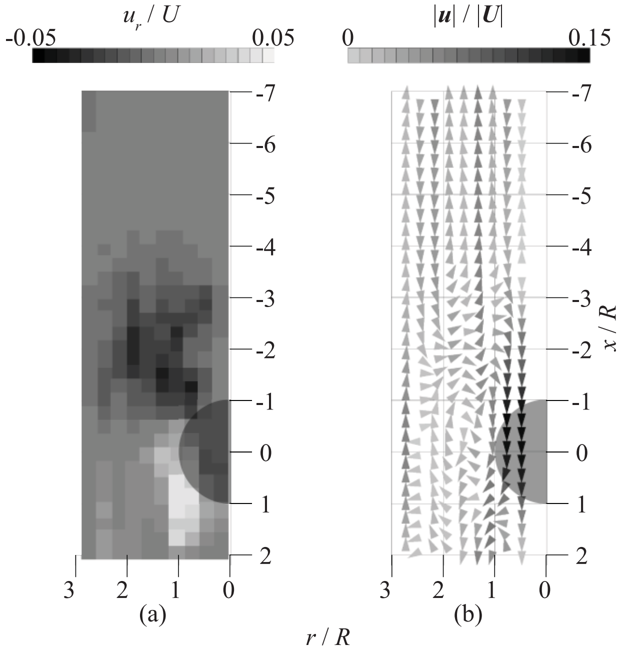

Figure 3: (a) Distribution of $u_r$ velocity component obtained with UVP (b) Velocity vector field obtained by applying the equation of continuity to Figure 3 (a)

## 5 DISCUSSIONS

### *5.1 On frozen hypothesis*

Figure 3 (b) is obtained under the assumption that sphere reaches terminal velocity. There are reports about oscillatory falling of a sphere in non-Newtonian fluids, and this must be checked in the present case for judging the validity of the frozen assumption. Figure 5 shows the time change of the position of the sphere, which is obtained from PIV. The vertical axis indicates the position of the sphere, $z$ at every time. The origin of $z$ corresponds to the initial position of the sphere. The dotted line indicates the position of UVP measurement line. It is determined from Figure 4 that the sphere reaches the terminal velocity around the measurement line. And more, it seems the flow field is steady in Figure 2. Therefore frozen hypothesis is applicable to the flow field.

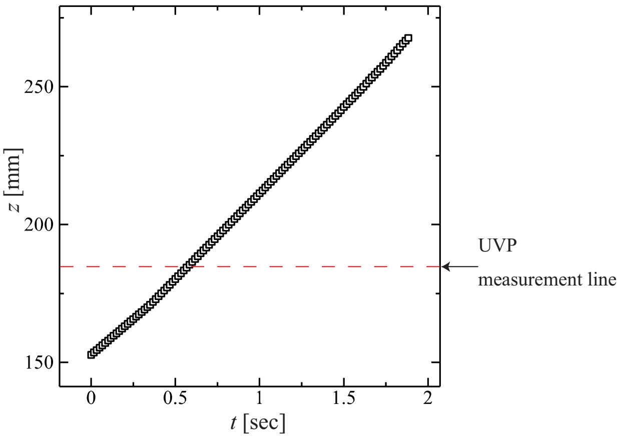

Figure 4: The position of the sphere in absolute coordinate

### *5.2 Equation of continuity*

Relevancy of applying the equation of continuity is verified by calculating the experimental divergence $\varepsilon$ defined as eq. (4).

$$\varepsilon = \frac{\partial u_x}{\partial x} + \frac{\partial u_r}{\partial r} + \frac{u_r}{r} \tag{4}$$

The equation of continuity is fulfilled when $\varepsilon = 0$. The spatial distributions of $\varepsilon$ both for PIV and UVP measurements are shown in Figure 5. The data in (b) are lacked near the center axis because of differentiation disabled. The magnitudes of $\varepsilon$ in Figure 5 (a) are much larger than those in Figure 5 (b). This fact indicates that the equation of continuity is unsuitable for the reconstruction of two-dimensional flow. One of the necessary conditions to regard the flow field axismmetry is that the sphere falls along the center axis of the cylinder precisely. A slight deviation of the sphere from the axis might lose the axial symmetry. This is the intrinsic problems when methodology of multi-dimensional velocimetry is built up at simplified equipment in falling sphere system.

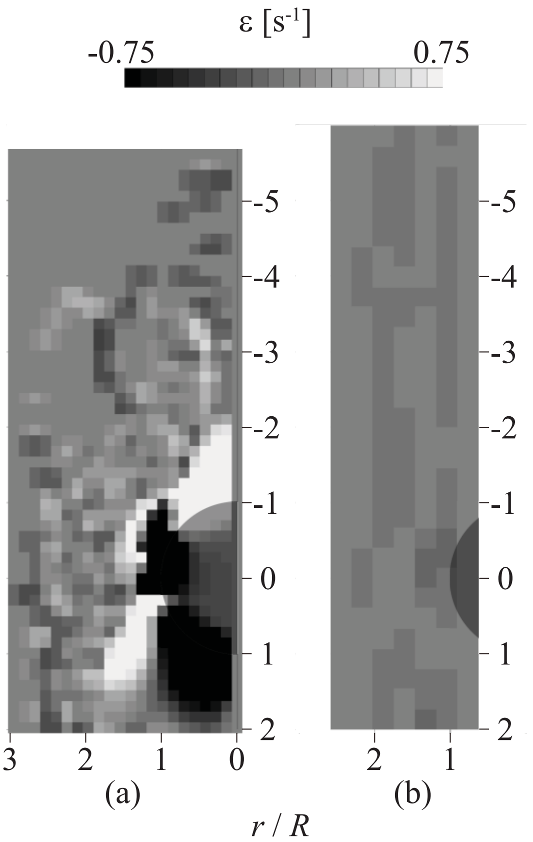

Figure 5: Distribution of $\varepsilon$ obtained from (a) PIV measurement (b) UVP measurement

### 5.3 Comparison of $u_r$ component

Comparison of the radial velocity distributions $u_r$ obtained by PIV and UVP is shown in Figure 6. It is noted that the measurement is done for different shot of the sphere, and thus the two results need not to be completely the same for the reasons mentioned in the former subsection. Nevertheless, a common structure is recognized in the two results, which is the potential displacement of flow upstream the sphere (shown by white region). The remarkable difference is seen in the location of inward flow (shown by dark color) behind the sphere. The reason of the diffference between PIV and UVP may be explained by reflection of ultrasonic pulse on the surface of the sphere. At the time around the sphere passes the UVP measurement line, measurement errors appear prominently. Futhermore, near the negative wake (around $x / R = -2.5$, $r / R = 1$, shown by glay region in Figure 6 (a)), the magnitude of $u_r$ is relatively small. Considering that the magnitude of velocity in Figure 6 (b) is smaller globaly than that in Figure 6 (b), it seems dificult to distinguish the negative wake from the errors shown above. This problem can be considered as a new fiding in UVP's application to viscoelastic flow.

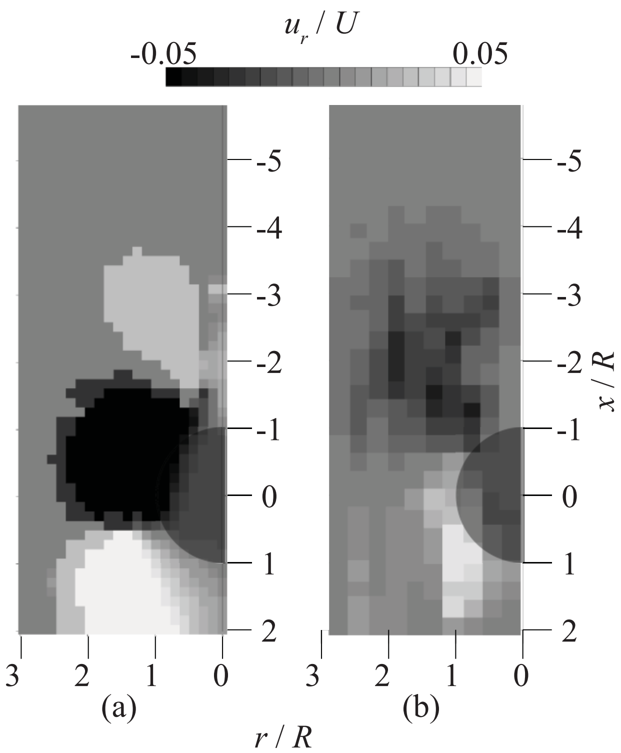

Figure 6: Distribution of radial velocity distribution $u_r$ obtained by (a) PIV, and (b) UVP

## 6 SUMMARY

A method of two-dimensional viscoelastic flow field measurement by means of UVP has been proposed. Key flow structures futuring the viscoelastic responce in falling sphere system have been discussed as the measurement performance was assessed. With the results, we have picked up two technical problems; magnitude of velocity near the negative wake, and departure from axial symmetricity in coupling equation of continuity. At present, we are now thinking that these problem will be overcome when multiple transducers are utilized simultaneously [3]. We are to examine this strategy in the next step of the study.

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