# Steady and unsteady flow characteristics of a shear thinning fluid through a collapsed elastic tube

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It was observed that for a given steady volume flow rate Q of a shear thinning (1.5 wt.% carboxymethyl-cellulose, CMC) aqueous solution, the tube was buckled from an elliptical shape to a line or area contacted two lobes as the critical external pressure $P_e$ increased. The downstream transmural pressure $P_{tm}$ was found to be more negative than that at the upstream as the outlet pressure decreased due to stronger tube collapse resulting in reduced cross sectional area. The tube cross sectional area decreased about an order of magnitude from the undeformed one when the compressive $P_{tm}$ was about 30 mbar at downstream, which was investigated using image analysis near the tube outlet at different $P_e$. The corresponding maximum flow velocity at the tube center increased about a factor of two as monitored using pulsed Ultrasound Doppler Velocimetry (UVP). A bi-modal velocity profile was observed in the two lobed tube shape from the plane above the horizontal axis of tube center by UVP. The periodic flow characteristics of CMC solution during ramp up and down were found to be different for the same range of Q, due to instantaneous inlet pressure response than that at outlet. The evolution of flow velocity profiles during the periodic ramp flow was measured and the velocity at the tube center increased by an order of magnitude as the time elapsed from 18s to 188s.

**Keywords:** bio-medical application, deformed elastic tube, intestinal flow, non-Newtonian fluid, steady and unsteady flow, transmural pressure, ultrasound Doppler velocity profile.

## 1 INTRODUCTION

The knowledge on the flow characteristics of non-Newtonian fluids in elastic inflatable and collapsible tubes is important to the biological applications especially for the understanding of the mixing and propulsion mechanism of food in small intestine during digestion. The mechanisms of pharyngeal, esophageal and intestinal transport of food and liquids are useful for the treatment of patients with malfunctioning of these transport processes [1]. Moreover, the flow behavior of non-Newtonian fluids in elastic tubes depends on the micro-structural properties of the fluids, solid mechanics of the elastic tube, the interaction between the deformation of the tube and fluids, and the applied stresses to induce flow [2]. The extensive experimental and theoretical contributions made by several authors [3,4,5] are mostly for the understanding of the complexity of the laminar and turbulent flows of Newtonian fluids through collapsible tubes together with the solid mechanics of the tube. However, there is little literature on the experimental flow characteristics of non-Newtonian fluids through elastic tubes under the influence of different transmural pressures and the corresponding velocity distribution in the collapsed tube. There are several publications on the velocity profiles measured using the ultrasound Doppler velocimetry (UVP) technique during steady and unsteady laminar flow of non-Newtonian fluids and particulate suspensions [6] in non-collapsible pipes. We have used [7] this technique to study the flow profiles of a non-Newtonian shear thinning solution during steady laminar flow through a partially collapsed elastic tube using Starling Resistor set up. In this paper, we investigate both the steady and unsteady laminar flow characteristics of shear thinning carboxymethyl-cellulose (CMC) aqueous solution (inelastic up to 2%) through a collapsible elastic tube under compressive transmural (internal minus external) pressures $P_{tm}$. As the external chamber pressure increased, the width at the tube center and cross sectional area of the deformed tube decreased; consequently the maximum flow velocity increased as measured by UVP. This technique was also used to study the temporal evolution of instantaneous flow velocity profiles during the periodic ramp flow in a collapsible elastic tube.

## 2 EXPERIMENTAL

### 2.1 Starling Resistor setup

The Starling Resistor is widely used [e.g. 5] to investigate flow through elastic tubes relevant to many applications. The experimental setup (Figure 1) in the present study consists of a cylindrical Plexiglass (PG) pressure chamber (300 mm inner diameter, 5.66 mm thick and 620 mm long) with two side metal flanges through each of which an aluminum pipe is fixed. A 20 mm inner diameter, 1 mm thick and 320 mm long silicone elastic tube (Lindemann GmbH, Germany) is mounted between the two aluminum pipes. The right pipe is connected to a rotor pump and a PVC tank containing the non-Newtonian liquid. Two pressure sensors (Sensor-Technics, Type: CTEM9350GY7) are installed on two PVC connectors for both the inlet and outlet aluminum pipes to measure the pressures of liquid.

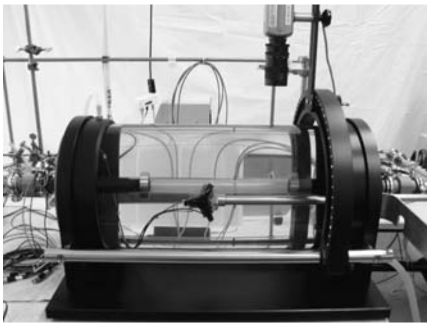

Figure 1: Photograph of the experimental setup for flow structure measurement of non-Newtonian fluids in elastic tubes using Starling Resistor.

One additional pressure sensor is connected to the pressure chamber to measure the externally applied pressure to deform the elastic tube. The three pressure transducers are connected to a data acquisition board (type USB-6221, National Instruments) with a resolution of 16 bits. A Canon PowerShot G2 camera with the resolution of 2272X1704 is used to obtain images of deformed elastic tube. The camera is mounted on an aluminum arm, which can be rotated to obtain images of the deformed tube at different angles. The steady and unsteady flows of a shear thinning non-Newtonian aqueous solution of 1.5 % carboxymethyl-celullose (Blanose CMC 7MF, IMCD Switzerland AG) at a range of volume flow rates and different values of the external chamber pressure, were carried out at 22°C.

### *2.2 Shear thinning solution*

In the present work, a shear thinning solution was used to investigate the flow structure of non-Newtonian fluids in elastic tubes. A carboxymethyl-cellulose (CMC) aqueous solution (1.5 % wt/wt with 0.1 M NaCl; $M_W$ = 2.5 ×$10^5$ g/mol) was used based on literature [8], which showed inelastic behaviour for concentrations up to 2%. Hence the rheological measurements of this solution at different concentrations were carried out using Physica (MCR 300, CC27) rheometer with concentric cylinder geometry (gap width = 1.13 mm).

### *2.3 UVP measurement in elastic tube*

A UVP-Duo (Met-Flow SA, Lausanne, Switzerland) instrument was used to measure the velocity profiles of CMC solution flowing through the elastic tube in the Starling Resistor setup.An ultrasound transducer with 4 MHz emission frequency and 5 mm active and 8 mm housing diameters (Met-Flow SA, Lausanne, Switzerland) was used as transmitter and receiver. One transducer arm together with holder to house the ultrasound transducer (for single or multiple transducer measurements) was immersed in the Perspex cylindrical pressure chamber filled with water. By adjusting the Doppler angle (20° with respect to the normal to the horizontal axis), the velocity profile during flow through the elastic tube can be measured using single transducer at different radial and axial locations of the (undeformed and deformed) elastic tube. The transducer submerged in water in the pressure chamber was in direct contact with the elastic tube without causing tube deformation. The communication with the UVP-Duo was made with an active X Library from Met-Flow SA. The number of cycles per pulse was 2 and the channel width was 0.38 mm. The measured velocity of sound in the CMC solution at 22°C was 1516 m/s.

### *2.4 Deformation measurement of elastic tube*

In order to determine the parameters during a steady flow of non-Newtonian fluid through the elastic tube, the shape and cross sectional area along the length of both the undeformed and deformed sections of tube were measured optically using images taken from different angles over the circumference.

A MATLAB based software was used to implement the computer tomography (CT) method for the 2D reconstruction of the elastic tube. The CT method was based on the backward projection of several images, which were obtained in different angles. The grid pattern was painted on the tube and pictures were taken by rotating the radially oriented camera around the tube shown in Figure 1. The angle between the pictures taken was known. Image processing (contrast maximization etc.) was applied to identify the grid lines drawn on the tube surface (left and center part of Figure 2). Then a rotation matrix (according the angle of the camera position) was applied to the images, which were combined (using addition or multiplication of the grayscale values). The projection beam lines for the lines on the tube cross in one point, which resulted in the position in 2D (right part of Figure 2). In addition to the CT-method, cell tracker software (also written in MATLAB) was used for the 3D reconstruction of the elastic tube. In this step, iteration was done over the tube length with a certain step change and several positions in 2D are constructed. Then, by using the cell tracker all the crossing points obtained at several positions in 2D were detected and 3D reconstruction was done.

## 3 RESULTS AND DISCUSSION

### *3.1 Rheology of CMC solution*

The measured shear rate $\dot{\gamma}$ dependent viscosities $\eta$ of different concentration CMC aqueous solutions were found to be shear thinning. 1.5 % CMC solution was further investigated, the measured (Figure 2a) viscosity of which was well represented by the Carreau model (Eq.1) with the fitted constants: $\eta_0 = 0.1452$Pa.s, $\lambda = 0.02673$ and $m = 0.7588$. The dynamic moduli measured under linear viscoelastic conditions by the oscillatory shear for 1.5 % CMC solution are shown in Figure 2b. The viscous (loss) modulus *G``* can be seen to be at least an order of magnitude higher than the elastic (storage) modulus *G`* indicating that the fluid is almost inelastic.

$$\eta = \eta_0 \left[1 + (\lambda \dot{\gamma})^2\right]^{(m-1)/2} \qquad (1)$$

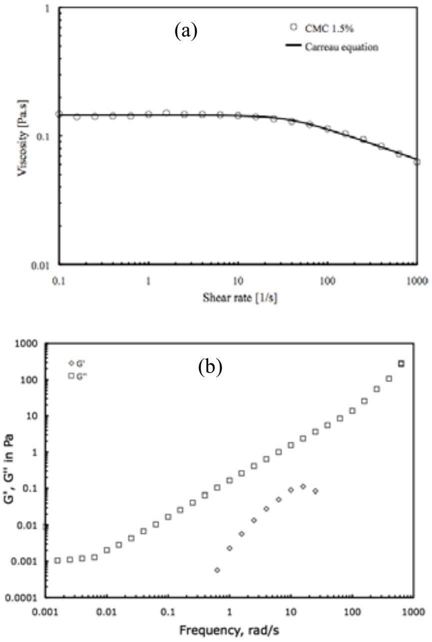


Figure 2: (a) Shear rate dependent viscosity (symbols: experimental; line: Carreau model) and (b) Dynamic moduli under oscillatory shear with a constant deformation of 5% at 22 °C for 1.5% (w/w) CMC solution.

### *3.2 Steady Flow of CMC solution in Collapsible tubes*

The flow behavior of non-Newtonian fluids in elastic tubes depends on the microstructural properties of the fluids, solid mechanics of the elastic tube, the interaction between the deformation of the tube and fluids [3], which includes the nonlinear pressure drop/flow rate relation [2]. Figure 3 shows that the pressure drop in the tube ($dP = P_i - P_o$) increases with increase in flow rate Q (17 ml/s) during steady flow of 1.5 % CMC solution up to a critical external pressure (about 95 mbar) due to increase of friction in uncollapsed tube. In contrast, above the critical external pressure the collapsed tube walls come closer and the pressure drop at a given Q increased as $P_e$ increased. On the other hand, when Q is further increased, internal pressure slowly increased and the tube recovered its original shape at higher flow rates decreasing the pressure drop. Moreover, the downstream transmural pressure was found to be more negative than the upstream one as the outlet pressure decreases when the tube collapses more strongly (Figure 4).

In addition, when the transmural pressure $P_{tm}$ is negative, the mechanics of the flow is closely

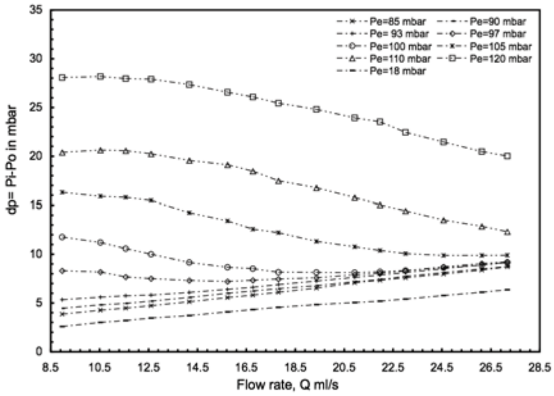


Figure 3: Influence of $P_e$ on the variation in pressure drop with flow rate .

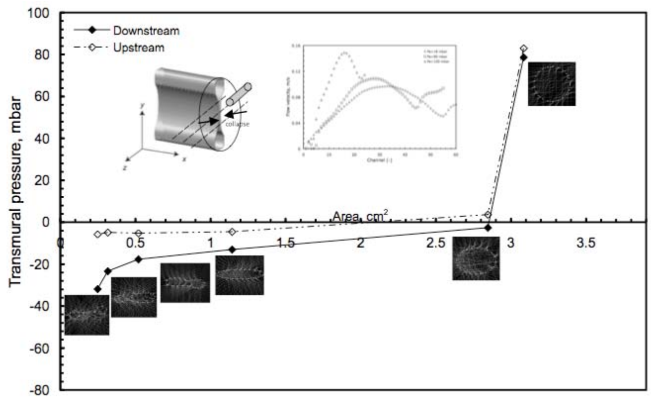


Figure 4: Transmural pressure versus tube cross sectional area during CMC solution steady flow in the elastic tube. The average maximum velocity versus area and schematic collapsed tube view is also inserted.

coupled to the structural mechanics of the tube and is characterized by the 'tube law': the relationship between cross-sectional area and transmural pressure [9] as shown in Figure 4. The tube cross sectional area decreased about an order of magnitude from the undeformed one when the compressive $P_{tm}$ was about 30 mbar at downstream and the corresponding maximum flow velocity at the tube center increased about a factor of two.

For a given flow rate (Q=17 ml/s) and an applied external chamber pressure of about 105 mbar lead to a change in the tube shape along the length. There was a section along the tube length where a two lobed shape was observed having a bi-modal velocity profiles for the corresponding tube shape as shown in Figure 5.

### *3.3 Unsteady Flow of CMC solution in Collapsible tubes*

The experimental flow characteristics of CMC solution under a periodic ramp flow ($4.7\times10^{-3}$ cycle/s and a flow rate amplitude of 13 ml/s) through elastic tube in the Starling resistor setup are shown in Figure 6.

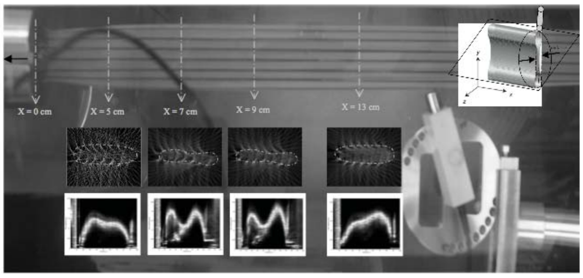


Figure 5: Variation in velocity profile,tube shape and cross sectional area along the tube length during steady flow of 1.5%CMC at 18 mbar downstream transmural pressure.

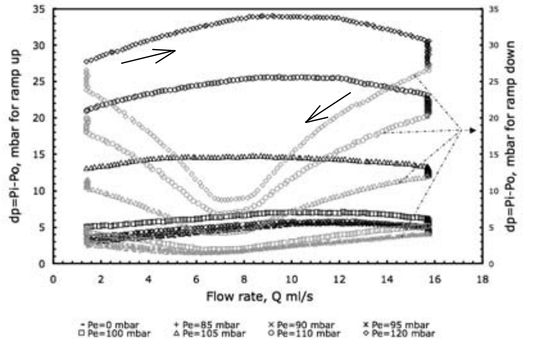


Figure 6: Variation in pressure drop with flow rate during periodic ramp flow of 1.5 % CMC solution through silicone elastic tube at different chamber pressures.

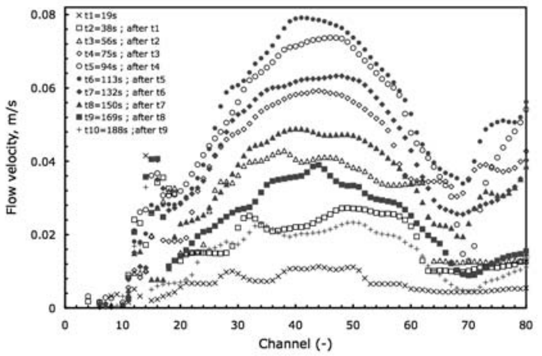


Figure 7: Temporal evolution of average flow velocity of 1.5 % CMC solution during a periodic ramp flow through elastic tube (open symbols: 10 $s^{-1}$ ramp up; closed symbols: 10 $s^{-1}$ ramp down).

In an unsteady flow (Figure 6) during the ramp up (10 $s^{-1}$), the flow characteristics of CMC solution can be seen to be similar to those of steady laminar flow for the same range of Q in section 3.2. In contrast, during the ramp down (10 $s^{-1}$) at higher applied external pressure (90 to 120 mbar), the pressure drop in the tube sharply decreases as Q decreases due to the instantaneous change of pressure response at the inlet than that at the outlet. Moreover, as Q reduces significantly, the tube walls come closer with a line/area contact of tube shape where the tube frictional force become dominant thereby increasing the pressure drop. Even for undeformed elastic tube, a higher average maximum velocity for the ramp down than that for the ramp up flow was observed. Figure 7 shows the temporal evolution of the average flow velocity at the tube center, which increased by about an order of magnitude during 1 cycle of the periodic ramp flow as time elapsed from 18s to 188s.

## 4 CONCLUSIONS

The steady and unsteady laminar flow characteristics of non-Newtonian shear thinning CMC aqueous solution through an elastic tube were investigated at different compressive transmural pressures in a Starling Resistor setup. The tube cross sectional area decreased by an order of magnitude from the undeformed one when the compressive $P_{tm}$ was 30 mbar at downstream and the corresponding maximum flow velocity at the tube center increased about a factor of two. A critical external chamber pressure caused collapse of elastic tube to form a two lobe-shaped cross-section, where measured UVP based velocity profiles are bi-model. The velocity profiles evolution during periodic ramp flow was measured and the velocity at the tube center increased by an order of magnitude as the time elapsed from 18s to 188s.

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