UVP measurement of low speed natural convection in the internally heating rectangular cavity

Tatsuya Kawaguchi
Department of Mechanical and Control Engineering, Tokyo Institute of Technology.
Addresses: Ookayama 2-12-1 Meguro Tokyo, Japan

Rayleigh-Bénard convection is one of the fundamental and significant flows of the chaotic and transitional fluid phenomena. The objective of the present study is to analyze a natural convection in a square cavity with Joule heating. Also the ultrasound velocity profiler (UVP) for the lower velocity fields was developed that enabled to measure the velocity below 1 mm/s. Cross correlation method in conjunction with an appropriate function fitting was employed to determine the local velocity. Temperature of the top of the cavity was fixed by the electrically insulated copper plate. The other five surfaces were adiabatic thermal boundary condition. A pair of opposed horizontal carbon plates was used as the electrode. The mixture of tap water and glycerol was used as a working fluid, LiCl was added as an electrolyte. Maximum Rayleigh number with internal heating was Ri~2.2x10^{10}. Velocity magnitude in the vicinity of the cooled surface was less than 10 mm/s, the disordered detachment of the thin thermal boundary layer was observed..

Keywords: natural convection, internal heating, cavity flow, cross correlation, cosine fitting.

1 INTRODUCTION

Natural convection with internal heat source is the flow due to the temperature difference within the enclosure. In many applications, the possible energy source of the internal heating is electric current for Joule heating, absorption of electromagnetic wave and such molecular or atomic reactions as chemical reactions and nucleorrhexis. For example, a mantle convection is one of the applications of the natural convection with internal heating[1]. Glass injection molding manufacturing as well as the vitrification of high-level radioactive waste is another industrial application of natural convection of molten grass with internal heat source. The numerical prediction and relevant experiments of the physics of a natural convection in a cavity has been investigated by many groups. Liaquat and Baytas[2] have numerically investigated the heat transfer characteristics of internally heated liquid in a cavity at high Rayleigh numbers from 10^7 to 10^{12}. They concluded that the flow pattern with lower Rayleigh number showed periodic oscillation whereas the higher Rayleigh number caused non-periodic time-dependent behavior. Since the instability of the flow field is essentially present and the considerable range of Rayleigh number is widely varied, the verification of each numerical model is complicated without the detailed and precise experimental data. Natural convection could be scaled by Grashof number, Prandtl number and Damköhler numbers in the presence of the internal heat source.

$$Ri = Pr \cdot Gr \cdot Da = \frac{g\beta q L^5}{k\nu\alpha} \tag{1}$$

Characteristic length, L, is equal to the size of the channel. q is calculated from the input energy density per unit volume. g represents gravitational acceleration. β, ν, α, and k are the thermal expansion coefficient, kinematic viscosity, thermal diffusivity and thermal conductivity of liquid respectively. The dimensionless parameter contains the intensity of the volumetric heating instead of a temperature difference between horizontal Dirichlet boundaries. Since the parameter increases directly with the fifth power of the characteristic length, the growth of the cavity size significantly affects the internal fluid behavior of cavity. The objective of the present study is to analyze a natural convection in a rectangular cavity with Joule heating by means of ultrasound velocity profiler. Moreover the improvement of the ultrasound velocity profiler was introduced for the sake of the measurement of the lower magnitude of velocity.

2 EXPERIENTS AND RESULTS

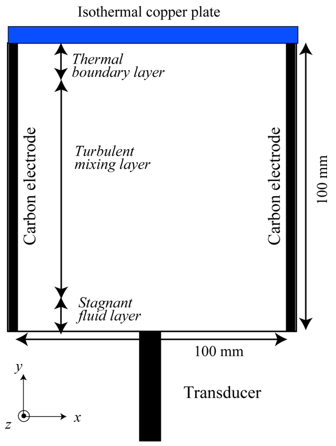


Figure 1. Simplified illustration of the experimental rig.

Figure 1 illustrates the simplified set up of the ultrasound measurement that consisted of a square transparent acrylic open channel with a pair of carbon plates as electrode, and a copper plate as a constant temperature ceiling. The size of the cubic cavity was 100 mm on a side, which is equal to L in equation (1). Copper plate with internal water channel was mounted on the top of cavity. The channel was connected to the heat exchanger with an automatic temperature control system. The temperature fluctuation of the plate was less than 0.1 K during the following experiments. The other five surfaces were assumed to be adiabatic by which the heat fluxes through the surfaces were equal to zero. The alternating current was applied for the energy source of the Joule heating. The frequency of the alternating current was 50 Hz due to the commercial electric supply. As the electric conductivity of the carbon graphite is 10^6S/m that is sufficiently larger than that of the working fluid, the electric potential in the carbon electrode was assumed to be constant. The 50 % mixture of glycerol and ion-exchanged water was used as a working fluid and LiCl was added as an electrolyte. The copper plate surface was protected by the insulating coating in order to avoid the undesired potential distribution in the test section. Sound speed of the glycerol and water mixture was c=1700 m/s that is assumed to be independent of the temperature of liquid. The arithmetic mean diameter of tracer particle was 0.1 mm. Relation between LiCl concentration and electric conductivity was calibrated in advance. Electric conductivity of working fluid was set to 0.2 S/m, mean electric current was 1.5 A. Consequently the density of internal heating by electric current was 135 kW/m^3 assuming that the electrical energy supply per unit volume was spatially homogeneous. Temperature of liquid was measured by the electrically-insulated thermocouples. Rayleigh number was 2.2x10^{10}. The resultant difference of local temperature between top and bottom of cavity was 40˚C.

For the velocity profile measurement, cross correlational ultrasound velocity profiler method[3,4] was used to obtain the fluid behavior of the natural convection. Conventional pulse Doppler ultrasound velocity profiler was also used. Velocity resolution of the latter system is 0.7 mm/s under the standard configuration that is not sufficient for the measurement of the natural convection with higher viscosity of liquid. Therefore the advanced cross correlation method was proposed that was explained in the following sentence. The frequency of the ultrasound was 4 MHz. For the echo signal monitoring, LeCroy 9360 storage oscilloscope was used. Simultaneously the PCI-bus connected high-speed 100 MHz analog to digital(A/D) convertor with on-board fast RAM was employed by which the triggered echo signal could be directly stored into the memory in computer. The captured consecutive echo signal was analyzed by the cross correlational technique. In the previous study[4], the aim of the development was the improvement of the temporal resolution. In contrast to the former study, the present contribution would focus on the extension for the low-speed measurement of the ultrasound technique. The present signal processing technique employed the appropriate infinite impulse filter as well as the new fitting function for the accurate determination of the sub-pixel displacement of sinusoidal waves.

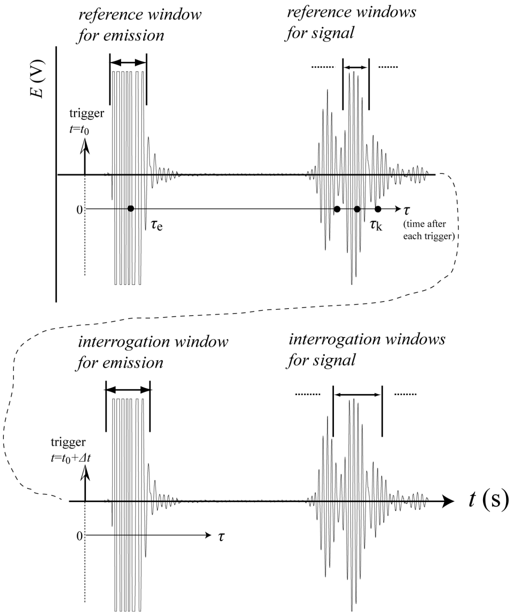
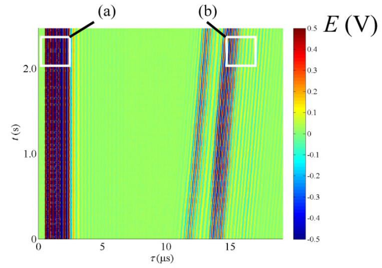


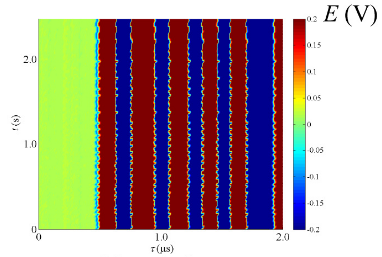
Figure 2. Definition of the post-trigger temporal axis, τ, and windowing scheme for the velocity distribution measurement by cross correlation technique. Not only the displacement of the reflected echo but also the fluctuation of the emission wave were considered between consecutive echo signals.

Figure 2 illustrates the example of the captured two successive echo signals. The definition of two temporal axes, t and τ, and the windowing position for the cross correlation technique was depicted. Each echo signal was consisted of the primary emission signal which is followed by the modulated sinusoidal echo. Since the input range of the analog to digital convertor was usually optimized for the echo signal that was extremely inferior to the emission signal, the emission signal was the rectangular shape due to the saturation of the converter. The DFT based coarse pre-processing followed by the fine direct cross correlation computation was performed. By the cross correlational UVP method, local velocity was determined by the displacement of the echo as $u = c\ \Delta\tau\ /\ 2\ \Delta t$. Therefore the calculation accuracy of $\Delta\tau$ directly affects the determination of velocity. The sizes of reference and interrogation windows for the real-domain correlational computation were 100 and

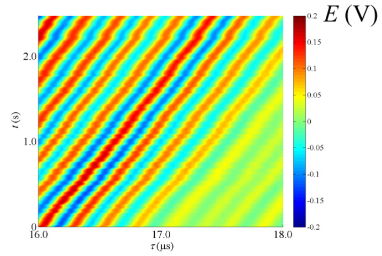
110 respectively. If the velocity magnitude of the fluid flow is sufficiently larger, the displacement of the local echo signal is several pixels, and the measurement error due to the sub-pixel interpolation is negligible. However the fluid investigation with very low velocity magnitude, the amount of the displacement is less than a pixel and the determination of the sub-pixel displacement is significantly important.



Entire echo signals



(a) emission



(b) reflected echo

Figure 3. The example of the captured intensity of the entire echo signals from the moving solid wall with constant velocity during 2.5 sec. The denotations (a) and (b) represent the magnified area of emission signal and reflected signal from the moving solid wall respectively.

Figure 3 depicts the acquired electrical voltage that is equivalent to the intensity of the received ultrasound. The horizontal axis is equal to the actual temporal domain, τ, of the digitizer. On the other hand, the vertical axis, t, corresponds the triggering intervals of the ultrasound emission, which is equivalent to the sampling frequency of the velocity profile measurement. This two-dimensional echo plot contains the entire information of the velocity profile along the measurement line. The denotation (a) and (b) in figure 3 shows the detailed echo signals of the emission region and of the reflected echo from the fluid flow respectively. Since the start trigger of the A/D convertor was electrically synchronized with the emission circuit of the ultrasound, the waveforms of the captured emission signal were quite similar and their phases were nearly fixed. However, the cycle-by-cycle instability of the emission signal is still remained that causes the bias error of the velocity profile. Although the noise reduction was partially considered as "emission trigger" in the previous study[4], the actual effect of the removal was not clarified. In order to determine the displacement of the reflected sinusoids accurately, the jitter noise was firstly estimated by the cross correlation methods with a triangular wave interpolation that is equivalent to the auto correlation function of the emission signal.

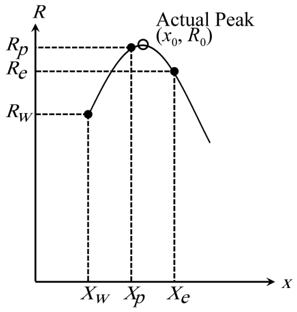


Figure 4. Sub-pixel estimation of the actual maximum of the cross correlation function by means of a function fitting.

The second improvement of the study is to employ the cosine function instead of the Gaussian curve for the sub-pixel interpolation. Figure 4 shows the fundamental scheme of the function fitting for the cross correlation coefficients. From the sampled echo signals by A/D converter, only the discrete correlation coefficients, R, are calculated. By using the neighboring three correlation coefficients, the actual peak location of function, x_0 could be determined by a function fitting. The cross correlation function of cosine waves must be cosine wave as well, not a Gaussian function but a cosine function is appropriate as follows.

$$x_0 = X_p - \tan^{-1}\left(\frac{R_w - R_e}{2R_p \sin(\omega)}\right)/\omega, \omega = \cos^{-1}\left(\frac{R_w + R_e}{2R_p}\right) \tag{2}$$

Since the fitting method could be applied with the arbitrarily-chosen three points to determine the peak location of correlation function, the least square value of the x_0 was calculated from the various combinations of the successive three coefficients. The multi-point peak estimation enabled to determine the precise peak location of cross correlation function and to determine the local velocity of fluid flow accurately as well.

As a preliminary experiment, the developed system was verified by the measurement of moving solid wall running at fixed low speed. Figure 5 compared the probability density distribution of measured 2000 velocity data. Relative velocity, u_{ref}, between transducer and solid wall as a reflector was varied, the emission interval of ultrasound, Δt, was considered as the parameter. When the velocity is equal to zero, the mean velocity was almost zero. The variation was, however, significantly affected by the change of Δt. With the smaller Δt, the value of $\Delta \tau$ is also small, therefore the relative error of the velocity determination was increased. The result showed that if the sampling interval of velocity profile is 100 ms, the standard deviation of velocity was 0.05 mm/s in the present configuration.

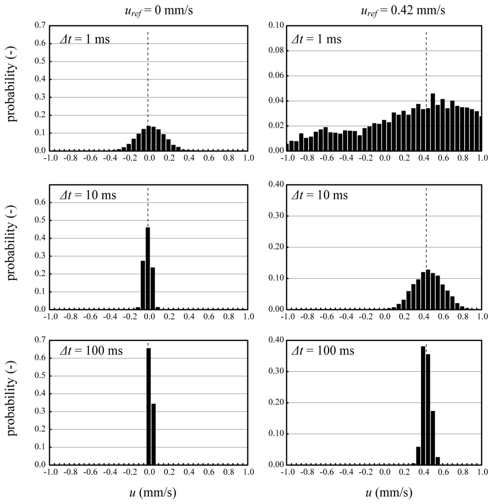


Figure 5. Comparison of the probability density functions of the measured velocity in terms of the frequency of the ultrasound emission. Dashed lines are the reference velocity of transducer at 0mm/s and 0.42 mm/s.

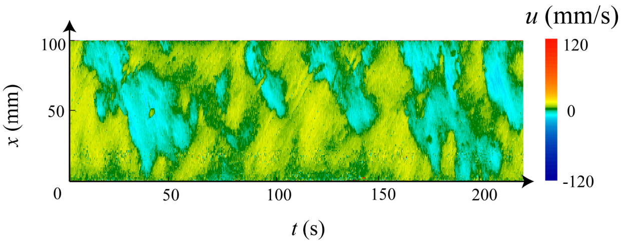


Figure 6. Velocity transition at the center vertical axis of the cavity during 220 sec by pulse Doppler technique. The upward and downward flows were alternately appeared. Cooled ceiling is located at x=100 mm.

The spatio-temporal velocity profile at the central vertical axis of the cavity was depicted in figure 6. The profile was obtained by the pulse Doppler method. The duration of the profile was 220 sec, sampling frequency of the velocity was 15 Hz approximately. The profile shows that the downward plume due to the detachment of the thermal boundary layer in the vicinity of the cooled surface was periodically occurred. However, no explicit cycle appeared in the term. Moreover the absolute value of velocity was quite small and it was difficult to analyze the detailed fluctuation of local velocity.

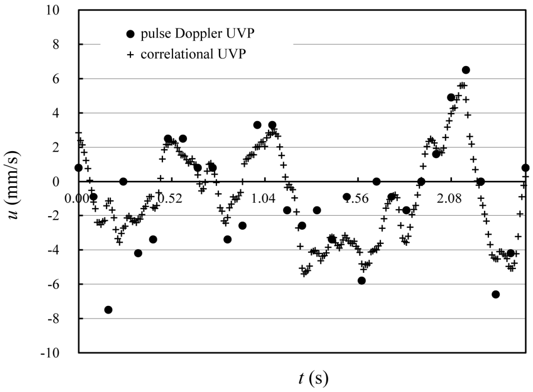


Figure 7. Comparison of measured velocity profile in the vicinity of the cooled surface at x=95 mm. Both conventional pulse-Doppler UVP system and present correlational techniques were compared. Resolution of the velocity was 1 mm/s by the pulse Doppler UVP, whereas that of the correlational technique was 0.05 mm/s.

Figure 7 compare the velocity profile obtained by the pulse Doppler method and by the advanced correlational techniques. Velocity profiles at x=95 mm were depicted. By the present measurement technique, not only the sampling frequency but also the velocity resolution was drastically improved and the detailed structure of the velocity fluctuation was obtained.

CONCLUSION

Natural convection in a square cavity with internal Joule heating was experimentally investigated by means of the advanced ultrasound velocity profiler. The flow field with the vertical temperature gradient due to the homogeneous internal heating and a low temperature ceiling is essentially unstable. The instability of the thermal boundary layer induces the characteristic downward flow in a cavity and the turbulent mixing around the center of cavity was significantly affected by the convective behavior of fluid. By the present technique, the standard deviation of the velocity measurement was 0.05 mm/s.

REFERENCES

[1] Turcotte D L and Schubert G: Geodynamics 2nd edition, Cambridge Univ. Press (2001)

[2] Liaqat A and Baytas A C: Heat transfer characteristics of internally heated liquid pools at high Rayleigh numbers, Heat and Mass transfer, 36 (2000), 401-405.

[3] William D B et al.: A New Time Domain Technique for Velocity Measurements Using Doppler Ultrasound, IEEE Trans. Biomedical Engng., BME-32, 3 (1985), 213-229.

[4] Ozaki Y et al.: High time resolution ultrasonic velocity profiler, Experimental Thermal and Fluid Science,26 (2002), 253-258.