# Uncertainty analysis of flowrate measurement using UVP

Noriyuki Furuichi

National Institute of Advanced Industrial Science and Technology (AIST), National Metrology Institute of Japan (NMIJ), Tsukuba Central 3, Umezono 1-1-1, Tuskuba, 305-8563, Japan

A fundamental uncertainty analysis of a flowrate measurement using UVP is carried out according to GUM. Dominant uncertainty factors are a resolution of a velocity and an inclination angle of an ultrasonic transducer. The expanded uncertainty whose coverage factor is 2 is estimated as 0.4%~1.0% depending on the inclination angle of a transducer. The actual flow calibration is also carried out in national standard of flowrate (NMIJ). The combined uncertainty of the calibration, deviation from reference flowrate and reproducibility is inside of the analytical uncertainty.

Keywords: Flowrate, Velocity profile, Calibration, Uncertainty

## **1 INTRODUCTION**

A study of a flowrate measurement using UVP has started by Takeda et al.[1]-[3] from about twenty years ago. Since the principle of this method is based on a direct integration of a velocity profile, a correction factor which is almost used for a commercial flow meter is not necessary. By this feature, this method can be applied to a flowrate measurement of many industrial fields without any influences of an upstream pipe condition such as an elbow, wall roughness and so on. In the field of metrology, this flowrate measurement is considered as most appropriate for a master meter for on-site calibration of flow meters installed in an existing pipeline. In that calibration, the uncertainty of the master meter (here, UVP method) is very important. In previous studies, accuracy of this method is considered as around 1% in theoretically[4][5]. Many actual flow calibration works have also carried out and deviations from the reference flowrate in each calibration are within about ±0.5%[2]. However, there is difficulty to obtain an actual accuracy of this method since a pipe condition is very different for each measuring target as case by case.

On the other hand, a fundamental uncertainty analysis for this flowrate measuring method by UVP has still not been carried out enough. It is very important to do the fundamental analysis of the uncertainty since those results might be necessary to calculate an uncertainty for the actual measurement. In this paper, the fundamental uncertainty analysis of a flowrate measurement of UVP based on GUM[6] is estimated.

Moreover, a calibration by a static weighing tank method is carried out to verify the uncertainty analysis. This calibration work is carried out in NMIJ's facility which is a national standard of flowrate in Japan. Under a careful arranged flow condition such as a fully developed flow, enough reflector and so on, the calibration is carried out for various flowrate and evaluate the uncertainty of the flowrate measurement by UVP method.

## **2 UNCERTAINTY ANALYSIS**

## 2.1 Analysis model of flowrate measurement

In this paper, the uncertainty analysis is carried out according to GUM[6]. GUM is recognized as the most global uncertainty analysis method as describing in international standard (ISO). In the metrology field, the uncertainty of a measurement device is normally estimated according to GUM.

Theoretically, flowrate is estimated by equations (1) -(3) which means an integration of a velocity profile as shown in Fig.1.

$$Q_{\rm U} = Q_{\rm C} + \sum_{i=1}^{72} Q_{\rm Ri}$$
 (1)

$$Q_{\rm C} = \frac{\pi \Delta r^2}{4} v_0 \tag{2}$$

$$Q_{\rm Ri} = 2\pi \Delta r v_i r_i \tag{3}$$

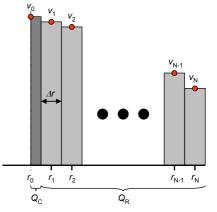


Figure 1: Integration model

Here  $Q_C$  is flowrate at the center of pipe,  $Q_{Ri}$  is flowrate at each measuring point,  $\Delta r$  is a split width,  $r_i$  is a radial position,  $v_i$  is velocity at each measuring point. Assumptions in this equation are following. The flow field in the measurement area is axisymmetric fully developed. The radial position described by  $r_N+\Delta r/2$  is completely fitted to the inner pipe wall. The pipe is completely circle. The velocity component of the radial direction is negligible small and an ultrasonic transducer alignment is immersion.

More assumptions to estimate an uncertainty are following. Measuring points are over 100[4]. Reflectors in the flow are satisfied to obtain a correct velocity. Then, an uncertainty of flowrate is estimated by the next equation.

$$u(Q_{\rm U}) = u(Q_{\rm C}) + \sum_{i=1}^{N} u(Q_{\rm Ri})$$
(4)

Here, the correlation coefficient is considered as 1. The relative standard uncertainty for  $Q_C$  and  $Q_{Ri}$  are described by next equations.

$$\frac{u(Q_{\rm C})}{Q_{\rm C}} = \sqrt{\sum_{j} \left(\frac{\partial Q_{\rm C}}{\partial x_{j}}\right)^{2} \left(\frac{u(x_{j})}{Q_{\rm C}}\right)^{2}} \qquad (5)$$

$$= \sqrt{\left(2\frac{u(\Delta r)}{\Delta r}\right)^{2} + \left(\frac{u(v_{0})}{v_{0}}\right)^{2}} \qquad (5)$$

$$\frac{u(Q_{\rm Ri})}{Q_{\rm Ri}} = \sqrt{\sum_{j} \left(\frac{\partial Q_{\rm Ri}}{\partial x_{j}}\right)^{2} \left(\frac{u(x_{j})}{Q_{\rm Ri}}\right)^{2}} \qquad (6)$$

$$= \sqrt{\left(\frac{u(\Delta r)}{\Delta r}\right)^{2} + \left(\frac{u(v_{i})}{v_{i}}\right)^{2} + \left(\frac{u(r_{i})}{r_{i}}\right)^{2}} \qquad (6)$$

Here, *u* means a standard uncertainty. As showing formula, uncertainty factors are A: split width and radial position, B: velocity.

### A. Split width and radial position

The split width and the radial position is calculated by next equations.

$$\Delta r = \frac{\Delta \tau}{2} C \cos \theta , \quad r_i = \frac{\tau_i}{2} C \cos \theta \tag{7}$$

Here,  $\Delta \tau$  is a delay time of pulse, *C* is a sound speed,  $\theta$  is an inclination angle of transducer. Since these equations are completely same, the uncertainty estimation will be only carried out for the split width. The relative standard uncertainty of the split width is estimated by the next equation.

$$\frac{u(\Delta r)}{\Delta r} = \sqrt{\left(\frac{u(\Delta \tau)}{\Delta \tau}\right)^2 + \left(\frac{u(C)}{C}\right)^2 + \left(\left|-\tan\theta\right|u(\theta)\right)^2 \quad (8)$$

The uncertainty of  $\Delta \tau$  is depending on the accuracy of the clock of the processer installed in UVP. Since the uncertainty of time measurement is relatively smaller than other factors, it is normally negligible.

The sound speed is set manually in UVP. Since the sound speed is a function of a temperature and a pressure, the uncertainty caused by the measurement of these values and the equation to estimate the sound speed should be estimated.

Moreover the uncertainty caused by a resolution should be included, when normal UVP such as UVP-Duo is used.

The uncertainty caused by the inclination angle of the transducer is very different whether the transducer setting is an immersion or a cramp-on method. In this paper, actual uncertainty estimation showing next section will be carried out for the immersion type to know the fundamental uncertainty.

### **B.** Velocity measurement

The velocity in UVP measurement is calculated by the next equation considering with the inclination angle of the transducer.

$$v_i = \frac{Cf_{\text{PRF}}}{512f_0} \frac{v_{\text{T}i}}{\sin\theta}$$
(9)

Here,  $f_{PRF}$  is a repetition frequency of a pulse,  $f_o$  is a basic frequency of the transducer,  $v_{Ti}$  is a digitized raw velocity component outputted from UVP. The relative standard uncertainty is following.

$$\frac{u(v_i)}{v_i} = \sqrt{\left(\frac{u(C)}{C}\right)^2 + \left(\frac{u(f_{\text{PRF}})}{f_{\text{PRF}}}\right)^2 + \left(\frac{u(f_0)}{f_0}\right)^2 + \left(\frac{u(v_{T_i})}{v_{T_i}}\right)^2 + \left(\left|-\frac{1}{\tan\theta}\right|u(\theta)\right)^2}$$
(10)

The uncertainty of *C* and  $\theta$  has already mentioned. The uncertainty of the repetition frequency and the basic frequency are based on the clock of a processor. As mentioned, these factors might be negligible.

In normal UVP, the velocity data is digitized to 256 steps. The uncertainty caused by resolution as shown in next equation should be included according to GUM.

$$u(v_{\text{Tinsti}}) = 0.5/\sqrt{3} = 0.29$$
 (11)

## 2.2 Uncertainty estimation for actual device

Since the flowrate measurement by UVP has so many parameters to measure the velocity profile, the uncertainty might be different case by case. Here, the measurement condition is set as Tab.1.

As a result of the uncertainty analysis, the budget sheet for 20° of inclination angle is shown in Tab.2. As mentioned, the uncertainty of the split width and the radial position is consists of one of the delay time, the sound speed and the transducer inclination angle. The uncertainty of the delay time is depending on the clock of the device. In a comparison of the frequency between calibrated frequency counter and UVP-Duo, any difference was not observed. Then, the uncertainty of the calibrated frequency counter is only used in this estimation. The uncertainty of the sound speed is caused by one of the temperature and pressure measurement and the resolution. The uncertainty of measurement of the temperature and the pressure is negligible small. In UVP-Duo, the sound speed is set manually with a step of 1 m/s. For the transducer angle, the standard uncertainty of the measurement of angle is estimated as 0.015° and the property of it estimated as 0.035° by considering as a linearity of an ultrasonic is 0.1%. The combined standard uncertainties of the split width and the radial position are estimated as 0.032%.

The uncertainty of the velocity is estimated as 0.247%. This uncertainty is relatively larger than other factors. Especially, a dominant factor of the uncertainty of the velocity is the raw velocity and the transducer angle. The dominant uncertainty source of the raw velocity is a resolution of it as shown in Eq.(11). When the ratio between the averaged velocity and the maximum one is set to 0.5, the standard uncertainty increases 0.224%.

Moreover, the contribution of the angle of transducer is very large in the uncertainty of the velocity. The standard uncertainty of it is estimated as 0.182% and this is the most dominant factor. The expanded uncertainty (k=2) of the transducer angle is estimate as 0.076° in this estimation. The contribution of the inclination angle increases with decreasing of it.

Expanded uncertainty for each inclination angle is shown in Tab.3. Since the uncertainty caused by inclination angle is dominant, expanded uncertainty is strongly influenced by it.

## **3 EXPERIMENT**

## 3.1 Experimental facility and pipe layout

An experiment was carried out using the national standard of flowrate in Japan. The national standard of flowrate is supplied using static weighing tank system[7]. A schematic diagram of the experimental facility is shown in Fig.2. Since water is supplied to the test section from the constant level head tank, a fluctuation of flowrate is relatively smaller than using pumps. Also, this calibration rig has a long straight pipe to achieve a fully-developed flow profile. In this experiment, the 250 mm test line and the 50 t weighing tank system are used. The standard uncertainty of the facility is  $u_{\rm C}$ =0.027% and the expanded uncertainty (*k*=2) of the calibration is 0.060%.

The detail of a test section is shown in Fig.3. From the upstream side, the test section is consists of a bubble generator, a flow conditioner, an over 50D straight pipe and a measurement device. As mentioned, the ultrasonic transducer is aligned as immersion to water.

In the following test result, the flow condition and the device condition including a setting parameter of UVP-Duo is according to Tab.1 and the inclination angle is  $8^{\circ}$ .

Table 1: Conditions for uncertainty estimation

Testing condition		
Water temperature	20 °C	
Pressure	0.2 MPa	
Flowrate	600,400,250, 170 m <sup>3</sup> /h	
UVP condition		
Number of measuring point	134	
Split width	0.74 mm	
Basic frequency	2 MHz	
Repetition frequency	256	
Averaged velocity / Maximum velocity	0.7	
Device condition		
Pipe diameter	198.85 mm	
Inclination angle	8°,12°,20°,30°	

Table 2: Uncertainty budget sheet for  $20^\circ$  of inclination angle of transducer

	Uncertainty Factor		Relative standard uncertainty
Split width		$\Delta r$	0.032%
Delay time		$\Delta \tau$	0.004%
Sound speed		С	0.021%
	Temperature		0.008%
	Pressure		0.000%
	Resolution		0.019%
1	TDX angle	$\theta$	0.024%
	Measurement		0.010%
	TDX property		0.022%
Radial position		r <sub>i</sub>	0.032%
Velocity		Vi	0.243%
5	Sound speed	С	0.021%
F	Repetition freq.	<b>f</b> prf	0.004%
Basic freq. Raw velocity		<b>f</b> 0	0.004%
		V <sub>Ti</sub>	0.160%
T	TDX angle	$\theta$	0.182%
	Measurement		0.073%
	TDX property		0.166%
Combined uncertainty		0.25%	
Expanded uncertainty (k=2)		0.49%	

Table 3: Expanded uncertainty for each inclination angle of transducer

Transducer inclination angle	Expanded uncertainty ( <i>k</i> =2)
8°	1.00%
12°	0.70%
20°	0.49%
30°	0.41%

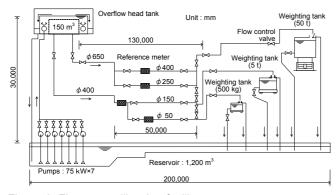


Figure 2: Flowrate calibration facility

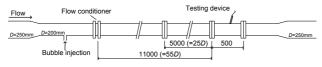


Figure 3: Test section

#### 3.2 Velocity profile

In order to check the velocity profile, UVP measurement was carried out on 6 azimuthal angles. In order to eliminate the error caused by a transducer setting or an installation of pipe, the test device with upstream pipe (25*D*) is rotated.

Velocity profiles are shown in Fig.4. Velocity profile at each angle show good agreement. Also, these velocity profiles are in good agreement with the power raw. It can be concluded that this flow field is completely axisymmetric fully developed flow.

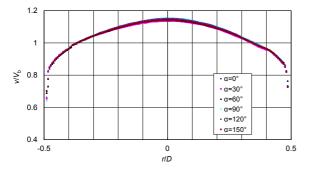


Figure 4: Velocity profile. Flowrate is 695 m<sup>3</sup>/s

### 3.3 Calibration result

Typical calibration results are shown in Fig.3. Vertical axis means a deviation from the standard value. Subscripts of UVP and ref. in figure mean a flowrate given by UVP and a reference flowrate obtained by the weighing tank. Error bars mean expanded uncertainty U calculated by the next equation.

$$U = k\sqrt{u_{\rm CMC}^2 + \sigma^2/n}$$
(12)

Here,  $\sigma$  is a standard deviation of the calibration result, *n* (=5, in this experiment) is a number for repetition, *k* (=2.4~2.7 in this experiment) is a coverage factor which is satisfied 95% confidential

level. In this examination, the expanded uncertainty is 0.14%~0.31% depending on the standard deviation of UVP measurement. Experiments were carried out at different three days to check reproducibility.

The deviation from the reference flowrate is result is less than 1% all over the measurement flowrate range. At flowrate 600 m<sup>3</sup>/h, it is 0.4% at most. This result is inside of uncertainty estimated in section 2. Reproducibility is 0.5% at most for all flowrate range and it is 0.32% at 600 m<sup>3</sup>/h. A combined uncertainty of the calibration, deviation and reproducibility is 0.51% at 600 m<sup>3</sup>/h. This uncertainty is inside of the analytical uncertainty.

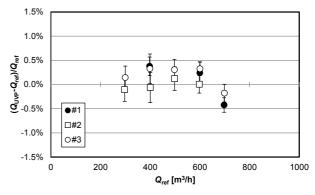


Figure 5: Calibration result

## **4 SUMMARY**

The uncertatinty of flowrate measurement using UVP-Duo is estimated as 0.4%~1.0% according to GUM uncertainty analysis. Dominant uncertainty factors are the resolution of velocity and the inclination angle of the transducer. The result of actual flowrate calibration using national standard is estimated as 0.51% for the analytical flowrate point. This value is inside of the analytical uncertainty.

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