Onset of oscillatory instability in Rayleigh-Bénard convection of a liquid metal layer under a horizontal magnetic field

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UVP is a good tool to visualize flow patterns in opaque fluids. We applied UVP to understand convection motion in a liquid metal layer affected by a horizontal magnetic field. Measurement lines of UVP arranged perpendicular to the magnetic field work to monitor quasi-two dimensional convection rolls in the liquid metal layer. Based on the spatio-temporal velocity distributions obtained by UVP and corresponding temperature fluctuations obtained by thermocouples, we widen the regime diagram of the convection for Rayleigh number, *Ra*, and Chandrasekhar number, *Q*. Variation of intensity for peak component on the power spectra of temperature fluctuations with decreasing *Q* at fixed *Ra* determined critical *Q* number for the onset of the oscillation on rolls.

Keywords: Ultrasonic flow visualization, Liquid metal, Convection, Flow instability

1 INTRODUCTION

The effect of magnetic field on a flow transition of liquid metal fluids is one of the interesting topics in fluid mechanics. Moreover, it has importance in geoscience in the point of thermal convections relating with geo-dynamo. For ideal unbounded fluid layers, applying a horizontal magnetic field does not provide any influence on the onset of the convection [1]. In actual bounded fluid layers, however, it stabilizes the fluid layer due to Hartmann breaking that is energy dissipation on the side wall. Then the critical Rayleigh number for the onset of the convection increases with increasing applying magnetic field [2]. If the magnetic field is strong enough, complex three-dimensional convection is stabilized into the two-dimensional convection rolls because of the effect of Lorenz force. In cases of moderate intensity of the magnetic fields, however, various flow patterns appear depending on a balance between Lorenz force due to the applying magnetic field and the buoyancy. In particular, spontaneous flow reversals are one of the active topics [3]. This phenomenon is accompanied by oscillatory instability and skewed-varicose instability on quasi-two-dimensional convection rolls [4]. But the appearance of these instabilities is not fully understood yet for the low Prandtl number fluids, because of opaqueness of the liquid metals: we cannot apply any optical flow visualization technique. To overcome this problem, our group has utilized Ultrasonic Velocity Profiler (UVP) to visualize convection motions in a liquid metal layer [3, 5]. The most recent publication provided regime diagram of the convection patterns including the difference of the number of rolls for Rayleigh number and Chandrasekhar number [4].

The present report is in the series of study in our group and focus on (1) widening the flow regime that was obtained in our previous study [4] and (2) investigating the onset of oscillation to understand the regime transitions of the convection. Busse and Clever predicted that the applying magnetic field modifies the onset of the oscillation, usually increases the corresponding critical Rayleigh number [6]. We investigated both of flow fields and temperature fluctuations to clarify the details of the flow transition influenced by bounded fluid layers.

2 APPERATUS AND METHOD

We used a square vessel under a horizontal magnetic field as shown in Fig. 1. The dimension of the fluid layer is 200 mm in sides and 40 mm in height, and corresponding aspect ratio is five. The side wall is made of polyvinyl chloride whose thickness is 30 mm to keep thermal insulation. The top and bottom boundary of the fluid layer are composed by copper plates. The temperature of each plate was kept constant by circulating water. A horizontal magnetic field was generated by a pair of rectangle shape coils which create 300 mT as maximum. The working fluid is gallium alloy, GaInSn, which melting point is -19℃ and has never solidified in a room temperature. The top and bottom surface of the fluid layer were kept at 20℃ as an initial condition. After giving a temperature difference in fluid layer, the horizontal magnetic field was applied. The flow patterns were visualized by UVP. The convective pattern in the liquid metal layer would be two dimensional roll structures when the applying magnetic field is strong enough. Therefore the line measurement by UVP is useful to investigate two-dimensional convection. The oscillatory instability appears as fluctuations of roll boundary on spatio-temporal velocity distribution obtained by UVP. However, at marginal conditions for the instability, the fluctuation cannot be distinguished because of very small fluctuation and relatively rough spatial resolution on obtained velocity distributions. To detect it, temperature inside the liquid metal layer was measured by thermocouples at points. The diameter of the each tip of thermocouple is 1 mm and measurement height is 5 mm below the top surface of fluid layer. Details of the measurement locations of three thermocouples are shown in Fig. 1 as labeled 5, 6, and 7. We also installed four other thermocouples to check the temperatures of top and bottom plates.

Thermal convection in this system is described by three non-dimensional numbers; Rayleigh number ($Ra=\beta g \Delta T L^3/\kappa\nu$) that indicates the balance between buoyancy and viscous force, Chandrasekhar number ($Q=\sigma B^2 L^2/\rho\nu$) that indicates the balance between Lorentz force and viscous force, and Prandtl number ($Pr=\kappa/\nu$) (2.7×10^{-2} in GaInSn). We controlled a temperature difference between the top and bottom surface of the fluid layer, ΔT and intensity of applying magnetic field, B. to modify Ra and Q, respectively. The range of Ra is from 10^4 to 10^5 and Q is from 0 to 2.3×10^5. In the corresponding range of Ra, the convection motion is highly developed thermal turbulence without applying magnetic field.

(a)

(b)

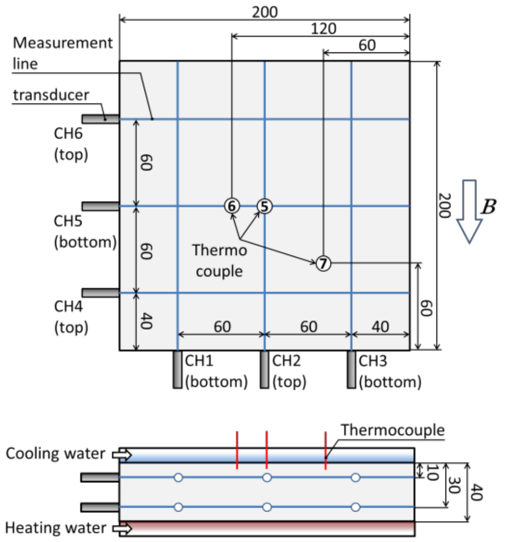

Figure 1: Schematic image of the experimental apparatus; (a) top view of the vessel and measurements line of velocity field by UVP method, and (b) side view

3 RESULTS AND DISCUSSION

3.1 Regime diagram

We performed the experiment at various parameters and made a regime diagram as shown in Fig. 2. The diagram includes our previous results that were already published [4], where the solid line in the figure represents the linear stability curve for the onset of the convection [2]: The critical value is strongly modified by applying magnetic field at large Q regions. In the condition of weak magnetic fields (smaller Q numbers) relative to Ra, the convection motion seems isotropic and we can distinguish that the flow is in thermal turbulence. On the other hand the stronger magnetic fields (larger Q numbers) relative to Ra modify the convection motion into quasi-two dimensional rolls. In the general tendency indicated in the regime diagram, stronger magnetic field (larger Q) increases the number of rolls. We identified 3 to 6 roll regimes in the present range of Ra and Q. The flow reversals mainly appear between the regimes having different number of rolls, 4 and 5, 3 and 4, as shown in the diagram. The regimes seem to be organized by the fraction of Ra to Q: Dashed lines in the figure indicate some constant fractions and the regimes in the region of low Q surely obey this rule. The rule is still roughly valid at high Q regions and there may be influences of modified critical Rayleigh number by Q.

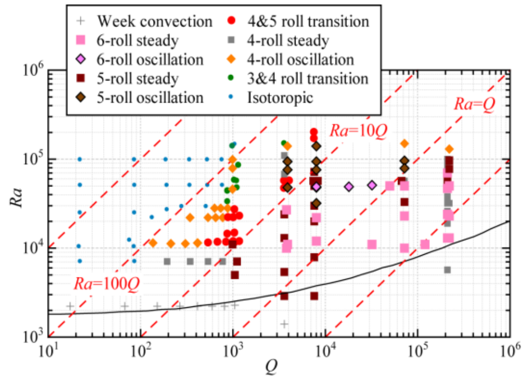

Figure 2: Regime diagram of convection behaviors including our previous results [4], where the solid line represents critical Rayleigh number modified by applying magnetic field estimated by Burr & Müller [2]

To focus on the onset of oscillations of quasi-two dimensional convection roll, Fig. 3 summarizes the transition of regimes with decreasing the magnetic field (decreasing Q) at the fixed temperature difference (fixed Ra). In the figure the flow patterns are categorized by the number of rolls that is distinguished on the spatio-temporal velocity distributions, and periodic fluctuations of temperature measured by thermocouples. This diagram gives us rough estimation of the parameter region for appearance of oscillatory convection. Fig. 4 shows spatio-temporal velocity distributions measured at ch-6 shown in Fig. 1 which is arranged perpendicular to the magnetic field. In contrast, the velocity components parallel to the magnetic field are much smaller than the velocity component shown here. Hence, these velocity distributions correspond to quasi-two dimensional convection rolls along the magnetic field with (a) 3 rolls, (b) 4 rolls, (c) 5 rolls and (d) 6 rolls, respectively. The schematic illustrations beside the velocity distributions in Fig. 4 show side views of estimated convection structure in the fluid layer. All distributions excluding (a) seem steady rolls. The number of rolls depends on *Ra* and *Q*. We have never observed steady 3 rolls in the present parameter range because buoyancy effect is much stronger than the effect of Lorenz force: *Ra* is least hundred times greater than *Q*. Excluding (a) in the figures, *Ra* is fixed and increasing of the number of rolls is realized by increasing *Q*.

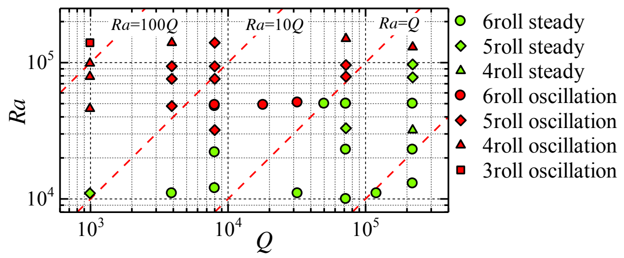

Figure 3: Regime diagram for convection behavior with decreasing *Q* at fixed *Ra*

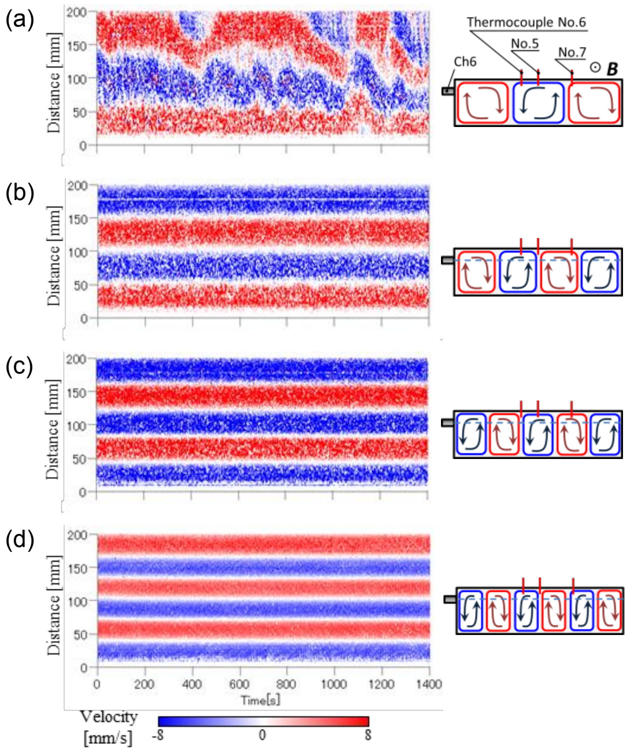

Figure 4: Spatio-temporal velocity distribution measured by UVP at Ch-6 indicated in Fig. 1, (a) 3 rolls (Ra~1.4×10^5, Q~2.3×10^4), (b) 4 rolls (Ra~5.0×10^4, Q~9.8×10^2), (c) 5 rolls (Ra~5.0×10^4, Q~3.9×10^3) and (d) 6 rolls (Ra~5.0×10^4, Q~2.3×10^5). The illustrations show side view of estimated convection structures.

3.2 Temperature fluctuation and onset of oscillatory instability

Fig. 5 shows temperature fluctuations corresponding to the velocity distributions shown in Fig. 4. These are measured at three points indicated in Fig. 1. The fluctuations (b) and (c) indicate that the convection motions are unsteady although the velocity distributions seem steady. The temperature fluctuations shown in Fig. 5(a) compose oscillations with various periods from short and long ones, and the amplitude of the fluctuations is much larger than others. The other fluctuations seem simpler: Fluctuations in Fig. 5(c) look quasi-regular sinusoidal waves. In the series of decreasing *Q* at the fixed *Ra*, the amplitude of the fluctuation generally decreases with increasing *Q*. But at each thermocouple the fluctuations dramatically change against *Q* without a simple tendency. This is because that the relative position of the thermocouple against the rolls changes with the transition on the number of rolls: Temperature fluctuation becomes large at the boundary of rolls and becomes small around the center of rolls. To characterize the fluctuations, we did spectrum analysis on the fluctuations as shown in Fig. 6. In the spectrum shown in Fig. 6(a), intensity gradually decreases from low frequency to high frequency, and there exist two slopes in high and low frequency regions. This is a typical property of the thermal turbulence. The spectrum shown in Fig. 6(b) has similar shape, but it has a broad peak around 0.06 Hz that corresponds to the main frequency of the fluctuation shown in Fig. 5(b). Two slopes cannot be distinguished in this condition and the intensity at high frequency regions relatively increases. In Fig. 6(c), the intensity concentrates on the main peak around 0.1 Hz and its harmonics that corresponds to the main frequency of quasi-regular sinusoidal fluctuation shown in Fig. 5(c). We cannot find clear peak on the spectrum shown in Fig. 5(d).

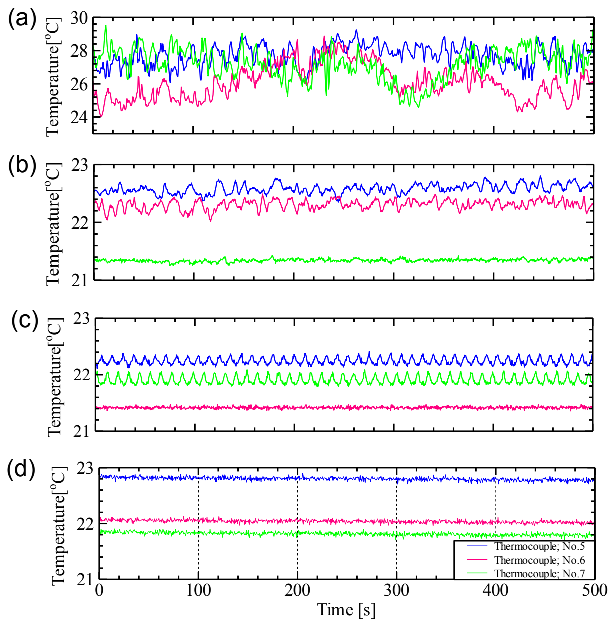

Figure 5: Temperature fluctuations in liquid metal layer at 5 mm below from the top surface at (a) Ra~1.4×10^5, Q~2.3×10^4, (b) Ra~5.0×10^4, Q~9.8×10^2, (c) Ra~5.0×10^4, Q~3.9×10^3 and (d) Ra~5.0×10^4, Q~2.3×10^5

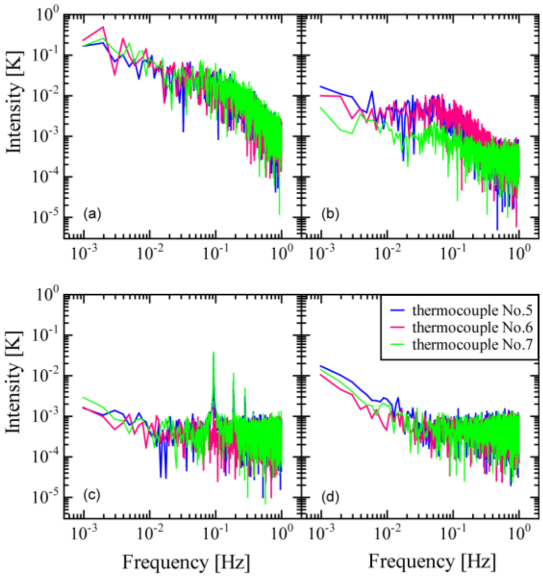

Figure 6: Power spectra of temperature fluctuations measured at (a) Ra~1.4×10^5, Q~9.8×10^2, (b) Ra~5.0×10^4, Q~9.8×10^2, (c) Ra~5.0×10^4, Q~3.9×10^3 and (d) Ra~5.0×10^4, Q~2.3×10^5

To determine the onset of roll oscillations, we focus on the flow pattern of six rolls at Ra~5.0×10^4 with variation of Q in 8.0×10^3 < Q < 2.3×10^5: This case provides most clear tendency of the variation and we introduce it as a typical example of the analyses. Fig. 7 (middle) shows the variation of the intensity that is extracted from peaks on the spectra with decreasing Q. The bottom figure indicates variations of frequency at the corresponding peak. Top figures represent examples of corresponding temperature fluctuations. The fluctuation obtained by the thermocouple No. 5 is very small and it does not provide clear tendency in the figures. For ther thermocouple No. 6 and 7, the fluctuations have clear peak frequency around 0.1 Hz and the frequency is almost constant for range of Q<6×10^4. In the corresponding range of Q, the intensity decreases with increasing Q and finally it reaches almost zero. The onset of the oscillation can be determined by extrapolation of this variation as zero-crossing point of Q. The obtained value is Q~1.1×10^5, and is the critical Chandrasekhar number for Ra~5.0×10^4. To continue the similar analysis on the different number of rolls, we would obtain stability curve for the onset of oscillation.

Inverse of the peak frequency provides typical time scale of the convection, and it becomes around 10 sec. This value is very close to the circulation time in the rolls, around 15 sec, that is estimated from typical velocity scale of the roll, 10 mm/s from Fig. 4(d), and perimeter of each roll, 2×(40 + 200/6) = 147 mm. This is consistent with our previous results.

4 CONCLUSION

The simultaneous measurements of velocity field and temperature fluctuation in liquid metal convection under a horizontal magnetic field were carried out to widen regime diagram of the convection and to determine the onset of oscillation on the quasi-two dimensional rolls. As the result, 3, 4, 5 and 6 rolls convection were observed in the spatio-temporal velocity distributions measured by UVP. Temperature fluctuations caused by oscillation of convection rolls appeared with decreasing the intensity of magnetic field at fixed temperature difference. We categorize the flow pattern into the transition regimes as shown in Fig. 3. This diagram provides rough sketch of the parameter region for appearance of the oscillatory convection. We focused on the transition from steady state to oscillatory convection at the 6-roll flow pattern. Variation of the peak intensity on the spectra of temperature fluctuations provided the critical Q number at the fixed Ra.

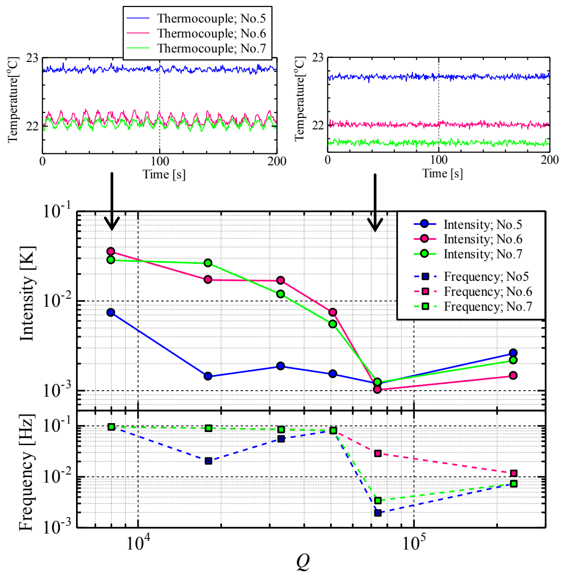

Figure 7: Variations of peak intensity in the spectra (middle) and frequency at the peaks (bottom) obtained at the condition of 6 rolls convection on Ra~5.0×10^4, where the top figures represent corresponding temperature fluctuations

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