Evaluation of measurement accuracy of a dealiasing method for use with ultrasonic pulsed Doppler

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Ultrasonic pulsed Doppler method has a limitation that both the maximum measurable velocity and the length cannot be increased at the same time. In order to overcome this limitation, the dual PRF (pulse repetition frequency) method and the feedback method have been proposed for measuring flow rate in a pipe. In this study, a rotating cylinder device was employed for evaluating accuracy of the velocities obtained by means of the dealiasing methods. Effects of the velocity extension number, the measurement volume and the number of pulse repetition for obtaining an instantaneous velocity profile on the uncertainties of the velocities are evaluated be comparing standard deviations of the velocities. It is shown that the extension number has an optimum value in each condition, and it should be set as small as possible to avoid the velocity aliasing. Furthermore, increasing size of the measurement volume is more effective for improving the measurement uncertainty in comparison to increasing the number of pulse repetition.

Keywords: Dealiasing, staggered trigger, dual PRF method, feedback method

1. Introduction

The ultrasonic pulsed Doppler method (UDM) is a useful technique for measuring one-dimensional velocity profile along the ultrasonic beam line. Thus, it has been utilized for measuring flow rate for integrating the velocity profiles over the pipe [1]. However, it is well known that the maximum measurable velocity and the length are limited by the Nyquist sampling theorem, and both them cannot be increased at the same time. Therefore, the greater the velocity in a large-diameter pipe is (*i.e.* the higher flow rate), the more difficult it is to measure the velocity using the conventional pulsed Doppler method. In order to overcome this limitation, several dealiasing methods have been developed [2,3].

The authors employed dealiasing methods, *i.e.* the dual PRF method and the feedback method, to measure velocities in a pipe. It was shown that the method made it possible to accurately determine flow rates six times greater than those that can be determined using the conventional UDM in a pipe [4,5]. However, size of the measurement volume was shown to be an important parameter in measuring velocities. With increases of the velocity, size of the measurement volume should be increased. However, the effects of the measurement parameters such as the measurement volume, range of the maximum measurable velocity on the measurement uncertainty has not been completely understood.

In this study, the target velocity and the measurement length are more than 3 m/s and 200 mm. Velocity measurements were carried out using a rotating cylinder device which covers the requested specification. The uncertainty of the velocity with the dealiasing method was investigated.

2. Dealiasing method

In the conventional UDM, multiple pulses are used to estimate the velocity of moving target [4]. Hence, it is well known that the maximum detectable velocity, $v_{\text{max,conv.}}$, and the maximum measurable length, L_{max} are expressed as;

$$v_{\max, \text{ conv}} = \frac{c}{4f_0 T} \quad , \tag{1}$$

$$L_{\max} = \frac{cT}{2} \tag{2}$$

,where c is sound speed in the medium, f_0 is the basic frequency of the ultrasonic pulses and $T = 1/f_{\rm prf}$ is the pulse repetition interval. Hence, it is impossible to increase the both $v_{\rm max,conv}$ and $L_{\rm max}$ at the same time.

A staggered trigger method is one of the dealiasing method. With this method, some pulse emission intervals are employed and velocities are calculated based on the phase shifts between the pulses. In this study, two pulse repetition intervals known as the dual PRF method was employed. Pulses are emitted at the intervals of T and $T+T_s$, the maximum detectable velocity, $v_{max,dual}$, is expressed as [4];

$$v_{\max, \text{dual}} = \frac{c}{4f_0 T_s} \quad . \tag{3}$$

Comparing the Eq.(3) with Eq.(1), it is known that the maximum detectable velocity can be increased with T/T_s times. It has been confirmed that the dual PRF method has large uncertainty in comparison to the conventional pulsed Doppler method by experiment. In order to improve the accuracy of the velocity estimation with the dual PRF method, the feedback method was proposed [4,5]. With the conventional method, if the velocity exceed $v_{max,conv}$ several times and the Nyquist folding

number, *m*, is unknown, the true velocity cannot be determined. On the other hand, with the feedback method, the *m* is corrected by the velocity which obtained using the dual PRF method. Velocity obtained by using the dual PRF method is used as a velocity index to determine the Nyquist folding number. Consequently, the velocity with the feedback method, $v_{\rm f}$, is obtained as

$$v_{\rm f} = v_{\rm conv} + 2mv_{\rm max, \, conv.} \,, \tag{4}$$

where v_{conv} is the velocity obtained using the conventional method. The *m* is estimated to satisfy following requirement:

$$v_{\text{dual}} - v_{\text{max}} < v_{\text{f}} \le v_{\text{dual}} + v_{\text{max}} , \qquad (5)$$

where v_{dual} is the velocity obtained using the dual PRF method and v_{max} is the maximum velocity based on the pulse interval. In the measurements, the same echo signals are used for the calculations in the conventional and dual PRF methods.

3. Experimental facilities

To evaluate the uncertainty of the velocity estimation, experiments were carried using a rotating cylinder. Schematic of the facility is shown in Figure 1. O is the center of rotation, d [m] is the distance between O and the measurement line, ω [rad/s] is angular velocity. If the rotating speed is X [rpm], ω is expressed as $\omega = 2\pi X / 60$. y is the distance from the inner wall surface along the measurement line, and Y is the half of length of measurement line in the cylinder. The inner diameter of the cylinder is 412 mm, and the cylinder height is 150 mm. d was set at 148 mm. Since the UDM obtains velocity components along the measurement line, if the rotated fluid and reflector can be considered as a rigid body rotation, the measured velocities along the measurement line are constant at d ω (= vtheory). The system can avoid any turbulence in the flow, and it allow to focus on the performance of the signal processing in



Figure 1: Experimental apparatus.

this ideal flow conditions. Sucrose aqueous solution with concentration of 4.7 wt.% was used as the working fluid to adjust the density with Nylon tracer particles (1.02 g/cc, 100 μ m) to avoid precipitation. The sound speed, *c*, is 1.52×10^3 m/s.

The experimental conditions were $v_{\text{theory}} = 1.550 \sim 3.100$ m/s which correspond to $X = 100 \sim 200$ rpm. *T* was constant at 5.56 ms, and the expansion number, $P (= T/T_s)$, was ranged between 9 and 40. Diameter of the ultrasonic beam was 10 mm with $f_0 = 2$ MHz. The number of pulse repetition for obtaining an instantaneous velocity profile, N_{pulse} , was 512 or 1024. Spatial resolution along the ultrasonic beam direction, ΔL , was 1.48, 2.22, 3.70 mm.

A laboratory-made measurement system [4,5] was used for the measurements. Reflected echo signals were recorded in a PC, and velocities were calculated after the measurements. 500 instantaneous velocities were used for calculating the time-average velocity profile.

4. Results and discussions

4.1 Time-average velocity profile

A time-average velocity profile obtained with the dual PRF method is shown in Figure 2. The rotating speed was 100 rpm and $v_{\text{theory}} = 1.550$ m/s. After 5 min from the start-up of the rotating cylinder, the measurement was conducted. It took approximately 20 min for measuring 500 profiles. The dashed line in the figure indicates $v_{\text{max,conv}}$. The velocity profile takes almost constant value in each measurement position except the near-side wall region. Thus, much higher velocity than $v_{\text{max,conv}}$ was accurately obtained with the dual PRF method. Furthermore, the velocity distributions can be considered to be rigid body rotation during the measurement.

Ultrasonic reflection on the wall surface of the rotating cylinder degraded accuracy of the velocity error in the near-wall region. Therefore, velocity data at the center, y/Y = 1, is used for the velocity evaluation.

4.2 Time-series Velocities

Time-series velocities with the dual PRF method and the feedback method at y/Y = 1 are shown in Figure 3. The rotating speeds are 100 and 150 rpm which correspond to $v_{\text{theory}} = 1.550$ and 2.325 m/s, respectively. The horizontal axis expresses the profile number. The other conditions



Figure 2: Time-average velocity distribution ($v_{\text{theory}} = 1.550 \text{ m/s}, 100 \text{ rpm}$)

are $\Delta L = 1.48$ mm, P = 9 and $N_{\text{pulse}} = 512$. Dashed lines indicate $v_{\text{theory}} \pm v_{\text{max}}$. Here, v_{max} is defined as;

$$v_{\rm max} = (v_{\rm max1} + v_{\rm max2})/2$$
 (6)

,where v_{max1} and v_{max2} are expressed as;

$$v_{\max 1} = \frac{c}{4f_0T},$$
 (7)

$$v_{\max 2} = \frac{c}{4f_0(T+T_s)}.$$
 (8)

As mentioned in the chapter 2, the feedback method employs the velocity with the dual PRF method, v_{dual} , to determine the Nyquist holding number. If difference between v_{dual} and v_{theory} is more than $\pm v_{max}$, misdetection of the Nyquist holding number that v_{max} becau. Hence, the values of $v_{theory} \pm v_{max}$ can be considered to determine whether the misdetection of the Nyquist holding number may occur with the feedback method.

Time-average velocities with the dual PRF and the feedback methods are tabulated in Table 1. It can be confirmed that the time-average velocities with the both methods are in good agreement with v_{theory} in each $0 \neq 0$ and 100.

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○ Dual PRF method \triangle Feedback method



(a) $v_{\text{theory}} = 1.550 \text{ m/s} (X = 100 \text{ rpm})$



(b) $v_{\text{theory}} = 2.325 \text{ m/s} (X = 150 \text{ rpm})$

Figure 3: Time-series velocities with the dual PRF method and the feedback method ($\Delta L = 1.48$ mm, $N_{pulse} = 512$)

Table 1: Time-average velocity with the dual PRF and the feedback methods

Vtheory	\mathcal{V} dual	$v_{ m f}$
1.550 m/s (100 rpm)	1.548 m/s	1.557 m/s
2.325 m/s (150 rpm)	2.327 m/s	2.324 m/s

However, variability of the velocity with the dual PRF method is strongly influenced by the measuring velocities. At $v_{\text{theory}} = 1.550$ m/s, the time-series velocities range within $v_{\text{theory}} \pm v_{\text{max}}$. On the other hand, velocities with the feedback method takes almost constant value at v_{theory} . It can be confirmed that the feedback method has lower uncertainty for the velocity estimation in comparison to the dual PRF method, and the feedback method could accurately estimate the velocities under such condition.

At $v_{\text{theory}} = 3.100$ m/s, the velocities with the dual PRF method include much error in comparison to those at $v_{\text{theory}} = 1.550$ m/s. Furthermore, time-series velocities with the feedback method take almost constant values at v_{theory} except at some profile number. If velocities with the dual PRF method are beyond $\pm v_{\text{max}}$ from the v_{theory} , the velocities with the feedback are misdetected.

4.3 Velocity Standard Deviation

The velocity standard deviations, σ , were calculated from the instantaneous velocities. The σ is theoretically zero if the measurement and experimental errors do not occur. Figure 4 represents the relation between v_{theory} and 2σ with different ΔL . *P* was set constant at 9 and the v_{max} was 0.321 m/s. Assuming that the velocity error is followed by the normal distribution, 95% of instantaneous velocities are ranging within $v_{\text{theory}} \pm 2\sigma$. Thus, if 2σ is smaller than v_{max} , the velocity has more than 95 % accuracy

 2σ with the dual PRF method, $2\sigma_{dual}$, takes around 0.14 m/s at 1.550 m/s. Therefore, the v_{dual} includes much error even though the measurement velocity is not so high. With increases of the measurement velocity, $2\sigma_{dual}$ gradually increases and it reaches v_{max} around $v_{theory} = 2.325 \sim 2.480$ m/s.

On the other hand, 2σ with the feedback method, $2\sigma_{\rm f}$, takes almost zero at $v_{\rm theory} = 1.550 \sim 1.860$ m/s for $\Delta L = 1.48$ mm, and the value rapidly increases at $v_{\rm theory} = 2.015$ m/s. When $v_{\rm theory}$ is less than 2.015 m/s, error of $v_{\rm dual}$ was not significant, and misdetection of the Nyquist holding number hardly occurs. It can be said that the velocity estimations can be accurately conducted under such conditions. If the Nyquist holding number differs by ± 1 , $v_{\rm f}$ varies $v_{\rm theory} \pm 2v_{\rm max}$ as shown in Figure 3(b). Thus, $2\sigma_{\rm f}$ rapidly increases although the $2\sigma_{\rm dual}$ gradually increases with $v_{\rm theory}$. Since $2\sigma_{\rm dual}$ for $\Delta L = 1.48$ mm is just beyond $v_{\rm max}$ at $v_{\rm theory} = 2.325$ m/s, it can be seen that 95% of $v_{\rm f}$ was accurately obtained under the condition.

 $2\sigma_{dual}$ for $\Delta L = 2.22$ mm is slightly lower than that for $\Delta L = 1.48$ mm. Thus, $v_{\rm f}$ could be obtained with 95% accuracy at $v_{\rm theory} = 2.480$ m/s for increasing ΔL to

0

1.5

2

foo

120 140 160 180 200 *X* [rpm]

Dual PRF method: $\bigcirc \Delta L = 1.48 \text{ mm}$ $\triangle \Delta L = 2.22 \text{ mm}$ Feedback method: $\bullet \Delta L = 1.48 \text{ mm}$ $\bullet \Delta L = 2.22 \text{ mm}$



Figure 4: Relation between the standard deviation and the rotating speed.

2.22 mm. This result indicates that large measurement volume is appropriate for measuring higher velocity.

Relation between $2\sigma_{dual}$ and *P* at $v_{theory} = 3.100$ m/s (X = 200 rpm) is shown in Figure 5. ΔL was set at 1.48, 2.22, 3.70 mm, and 512 or 1024 pulses were used for a velocity estimation. v_{max} depends on *P* which is related to T_s . Therefore, the relation between v_{max} and *P* can be obtained from Eqs. (6)–(8), and it is expressed as;

$$v_{\max} = \frac{c(2P+1)}{8f_0T(P+1)} \qquad (P>1) \tag{9}$$

Eq. (9) is indicated as a dashed line in the figure.

For $\Delta L = 1.48$ mm with $N_{\text{pulse}} = 512$, $2\sigma_{\text{dual}}$ takes minimum value at P = 18. Please note that the $2\sigma_{dual}$ at P = 30 and 40 were much large and these were over ranged in the vertical axis. If the P is smaller than 18, the velocity aliasing was caused in the velocities with the dual PRF method because of low accuracy of the velocity estimation. On the other hand, effects of the uncertainty of the measurement system such as T, T_s and noise effects become significant with increasing P. The appropriate P were 13 for $\Delta L = 2.22$ mm and 12 for $\Delta L = 3.7$ mm. It can be said that the appropriate P exists in each condition and it should be determined as low as possible not to occur the velocity aliasing. The $2\sigma_{dual}$ at appropriate P decreases with increasing of ΔL . Difference of $2\sigma_{dual}$ between at $\Delta L = 1.48$ mm and 2.22 mm is significant. In addition to ΔL , N_{pulse} is an important parameter for the velocity estimation. $2\sigma_{dual}$ decreases with increases the $N_{\rm pulse}$ although the velocity time-resolution becomes worse. $2\sigma_{dual}$ with the same ΔL and P is roughly proportional to $1/\sqrt{N_{pulse}}$, and the difference of $2\sigma_{dual}$ for $N_{\text{pulse}} = 512$ and 1024 is not significant. Hence, increasing ΔL is effective to reduce the velocity uncertainty in comparison to increasing N_{pulse} . However, $\Delta L = 3.7 \text{ mm}$ with $N_{\text{pulse}} = 1024$ was required for measuring velocities with 95% accuracy at $v_{\text{theory}} = 3.100 \text{ m/s}$ (X = 200 rpm).



Figure 5: Effects of *P*, N_{pulse} and ΔL on the velocity estimation. ($v_{\text{theory}} = 3.100 \text{ m/s}$)

5. Summary

In order to evaluate the uncertainty of velocity estimations with the dual PRF and the feedback methods, basic experiments using a rotating cylinder device were carried out. The velocity estimation with the dual PRF method includes larger uncertainty in comparison to that with the conventional pulsed Doppler method, *i.e.* single PRF method. Thus, the feedback method is appropriate for the dealiasing. Accuracy of the feedback method could be evaluated with the standard deviation of velocity with the dual PRF method. If $2\sigma_{dual}$ is lower than v_{max} , the velocities have more than 95% accuracy. *P* has appropriate value in each condition. In order to increase the accuracy of the velocity estimation, increasing the ΔL is more effective than increasing the N_{pulse} .

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