Measuring error estimation of the ultrasound array flow mapping system by means of numerical simulations

Sven Franke[1] and Sven Eckert[1]

[1] Helmholtz-Zentrum Dresden-Rossendorf, Institute of Fluid Dynamics, Department Magnetohydrodynamics, P.O. Box 510119, 01314 Dresden, Germany

A new two-dimensional ultrasound Doppler flow mapping system based on the application of linear arrays has been developed recently. A main feature involves a multi-beam operation facilitating a high frame rate. Previously, the effect of crosstalk between the beams was investigated in a rotational flow by comparing the results of multi- and single-beam operation with each other. However, due to slight variations in the flow conditions and the scattering particle distribution the determined systematic error of measurement was not very reliable. Likewise, flow phantoms suffer from a number of shortcomings as fluctuations of rotational speed of the phantom drive or inadequate parameters of scattering particles. For this reason, we developed a numerical model of our flow mapping system providing the echo signals of the particle motion in a model flow being similar to our typical small scale experiments. For each particle the scattering signal is calculated by solving the Rayleigh integral by means of systems theory and summed to the total echo signal. This task was performed by the *FieldII* toolbox for *MATLAB*. In our paper we will present a detailed analysis of the systematic error depending on the flow structure. The error of the multi-beam mode in comparison to the single beam operation will be estimated.

Keywords: Ultrasound array, flow mapping, numerical simulation of ultrasound systems, FIELD II

1. Introduction

The pulsed ultrasound Doppler method has proved as a reliable and attractive flow measuring technique for non-transparent fluids including liquid metals [1]. A detailed study of unsteady multi-dimensional flow structures often requires the measurement of instantaneous flow velocity fields instead of merely velocity profiles. However, previous approaches of flow field measurements with the ultrasound Doppler method suffer from a lack of sufficient spatial and temporal resolution for such studies [2]. Recently, an enhanced measurement method for a multidimensional transient flow mapping based on the application of linear ultrasonic transducer arrays was developed [3] to overcome these drawbacks by implementing specific array driving techniques.

A main feature providing the high mapping rates deploys simultaneous measuring ultrasonic beams. However, this multi-beam operation induces a bias in the measured velocity of the flow map by acoustic crosstalk between the ultrasound fields. An experimental study revealed a sufficient small systematic error [4] of the multi-beam measurement compared to a velocity profile measured only with a single beam. However, this approach suffers from different issues: the reference flow (in a liquid metal driven by a rotating magnetic field) was not strictly steady-state and the velocity bias depends on the flow structure. Other flow phantoms as a rotating cylinder (filled with a particle-water mixture [1]) corresponding to a rigid-body motion also lacks in precision as a result of variations in mechanical and electrical components of the flow model setup. Furthermore random errors caused by the statistical distribution of the scattering particles as well as electrical noise could not be replicated for other conditions in a parametric study.

For these reasons a numerical approach based on linear acoustics for the investigation of velocity deviations and bias of the flow mapping system was chosen. After providing a brief outline of the system's principles the numerical method as well as the simulation model including the signal processing will be explained. Finally, some of the simulation results will be presented.

2. Ultrasound flow mapping system

The flow mapping method is based on the application of linear ultrasonic transducer arrays. The current technical implementation applies linear arrays composed of 25 plane transducer elements (transmission frequency 8 MHz) of 2.3 × 5 mm² with an element pitch of 2.7 mm (Fig. 1) which spans a measuring field length of 67 mm. A single array facilitates the measurement of the flow velocity component perpendicular to the transducer surface. Multiple array arrangements facilitate e.g. the measurement of both in-plane velocity components in a field or the measurement of several planes side by side.

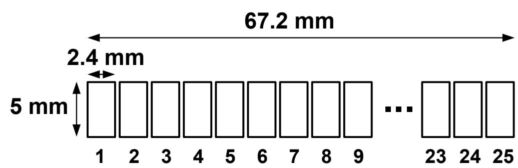

Figure 1: Configuration of linear transducer array

A specific electronic traversing scheme and pulsing strategy promote enhanced spatial and temporal resolution capabilities. An improvement of the spatial resolution is achieved by the application of the segmental array principle: During operation two adjacent transducer elements are interconnected to operate as one aperture of approx. 5 × 5 mm² reducing the beam divergence over the fixed measuring depth. The active transducer aperture can be traversed by one pitch length. This additionally facilitates the measurement of intermediate velocity lines thereby taking account of the self-focusing effect of plane apertures which makes the ultrasonic beam width closer than the aperture size over a specific depth.

Two enhanced approaches are applied to extend the temporal resolution: The first method implements a multi-beam operation which targets to scan as many profiles measuring lines (respectively transducer pairs) simultaneously as possible, thereby taking into account a sufficient small acoustic crosstalk. The second method is related to the pulsing strategy: As generally known the pulse repetition frequency of the ultrasound Doppler method is selected according to the velocity measuring range as well as the measuring depth. For small scale experiments with moderate flow velocities the time required for recording one echo signal (according to the measuring depth) is much lower than the pulse repetition time inducing an idle time between end of one echo acquisition and the begin of the following one. Contrary to previous multiline systems our approach applies this idle time for the echo acquisition of further measuring lines according to the multiplex pattern in Fig. 2 (in combination with the multi-beam and segmentation operation). Prerequisite is a fast changeover between the transducer channels by means of fast electronic switches.

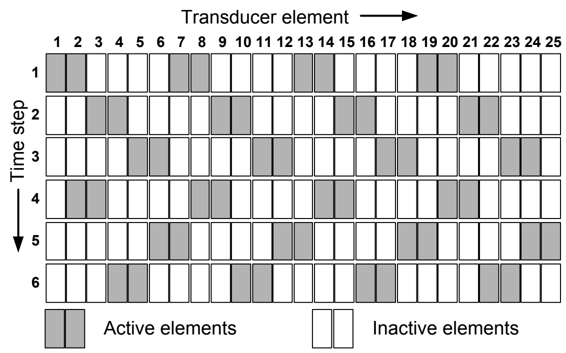

Figure 2: Pulsing scheme of a single array

3. Simulation of the ultrasound system

3.1 Numerical methods

The Rayleigh integral as a surface integral of an acoustic aperture allows the calculation of the acoustic pressure field at arbitrary points in space. This integral can be expressed in terms of linear system theory where every field point is described by an individual spatial impulse response for arbitrary aperture geometries. One method for calculating spatial impulse response is to integrate over the bounding lines of the aperture which will be applied in this paper due to its high accuracy. Another algorithm divides the aperture into small rectangle sub-apertures and summing up the far field approximations of all sub-apertures.

The interchangeability of acoustic source and receiver also permits the calculation of the pulse-echo field. The convolution of transmit and receive spatial impulse response (which may be assumed to be the same if the transmitting aperture is also the receiving aperture), electromechanical impulse response of the transducer and voltage excitation signal results in the received signal from a point scatterer. The echo signal of a collection of randomly distributed scatterers is obtained by summing up the individual scattering signals. If these point scatterers are shifted according to a velocity field and for every pulse emission the echo signals are calculated the records of an ultrasound flow mapping system are simulated. Please take into account that this simulation model only considers the echo displacement but not the Doppler shift of the particle motion in the received signals.

The entire simulation chain is implemented in the *MATLAB* toolbox *FieldII* provided by Jensen et al [5, 6]. All simulations are carried out using this toolbox.

3.2 Flow model

Typical flow configurations measured with the flow mapping system are vortex structures in closed vessels e.g. flows generated by traveling and rotating magnetic fields [7]. A well-suited stationary flow model for such vortex cells is the Roberts flow [8]. For simplification of this simulation approach only a two-dimensional modification is applied:

$$\begin{aligned} v_x &= -v_0 \sin x \cos y \\ v_y &= v_0 \cos x \sin y \end{aligned} \tag{1}$$

An example for a Roberts flow with 2×2 vortex cells is given in Fig. 3. Please note that there is no no-slip condition at the boundaries of the flow model; instead an infinite flow field is assumed.

The particle trajectories of the point scatterers (with their random distribution as initial condition) are determined by solving the pair of ordinary differential equations of Eq. 1 in terms of the numerical Runge-Kutta method.

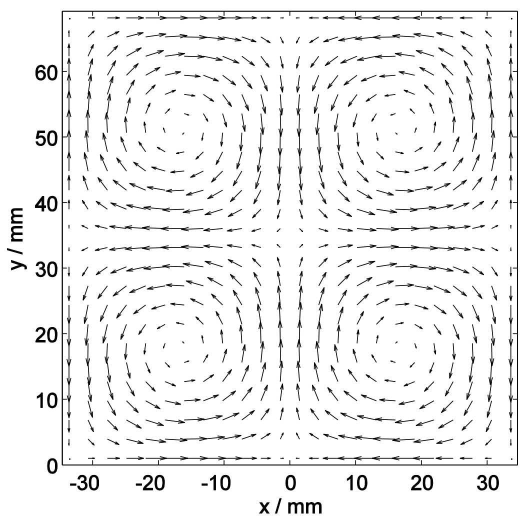

Figure 3: Velocity field of Roberts flow with 2×2 vortex cells as flow model for the ultrasound simulation

3.3 Signal processing

For accuracy reasons the simulation is carried out with a very high sampling frequency. However, for the signal processing the echo signals are sampled down to typical sampling rates. The velocity estimation of the simulated ultrasound echoes is performed by the standard algorithm according to Kasai [9]. The matched filter is adapted to the number of cycles of the sinusoidal excitation pulse. The application of a clutter filter was dispensed with since the modelling of stationary echoes as a result of multiple wall reflexes was omitted. Since the randomly distributed point scatterers give rise to a variance of the measured flow velocities always large sets of echo data are calculated. The determined velocity data of these sets are averaged to obtain a mean velocity profile free from statistical deviations.

3.4 Simulation parameters

The simulation parameters reflect the actual measuring system: The numerical model of the transducer array complies with the real array geometry from Fig. 1. The sound velocity is chosen according to the liquid metal GaInSn usually deployed at our experiments. The edge lengths of the simulated velocity field correspond to the aperture length. The parameters of the simulation are presented in Table 1 and the sinusoidal excitation pulse is shown in Fig. 4.

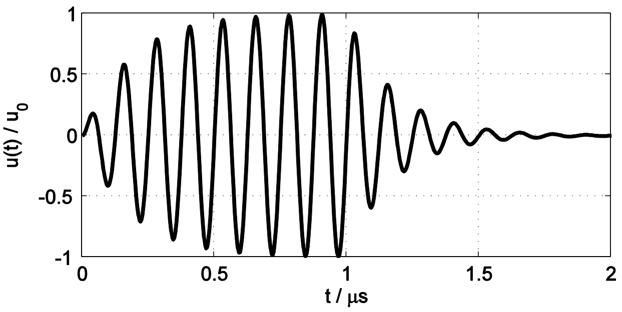

Figure 4: Excitation pulse of 8 sinusoidal cycles with bandwidth limitation to model the measurement electronics

Table 1: Simulation parameters

Parameter	Symbol	Value
Transducer frequency	f_0	8 MHz
Sound velocity (GaInSn)	c	2740 m/s
Number of cycles (excitation pulse)	N_c	8
Flow field direction 1	s_x	67.2 mm
Flow field direction 2	s_y	67.2 mm
Field thickness	s_z	20 mm
Peak flow velocity	v_0	50 mm/s
Particle density	ρ_s	0.1 mm^{-3}
Field II sampling rate	f_S	250 MHz
Echo sampling rate	f_A	25 MHz
Pulse repetition frequency	f_{PRF}	1217 Hz
Emissions per profile	N_{EPP}	50
Velocity range	v_{max}	104 mm/s
Gate distance	d_{Gates}	0.685 mm
Simulated echo signals per flow configuration	N_E	300 000

4. Results

A study with manifold parameter variations is performed focused on the analysis of the systematic error of measurement (bias) induced by the acoustic crosstalk of the multi-beam operation. Particularly the underlying flow structure of the simulation is varied with a different number of vortex cells (always same number in both field directions) within the measuring field up to 6×6 cells.

The ultrasound echo signals are simulated once for a conventional measurement with a single beam and once in multi-beam operation (see Fig. 2). A result for two different flow configurations is shown in Fig. 5. The results reveal that the acoustic crosstalk induces a significant bias of the velocity profile. Moreover the bias rises with increasing measurement depth as a result of increasing beam width induced by the beam divergence. This effect is especially exposed in flow configurations with smaller cells. Also the measured profiles of the conventional single-beam measurement exhibit a bias resulting from the limited spatial resolution capabilities of the ultrasonic pulse.

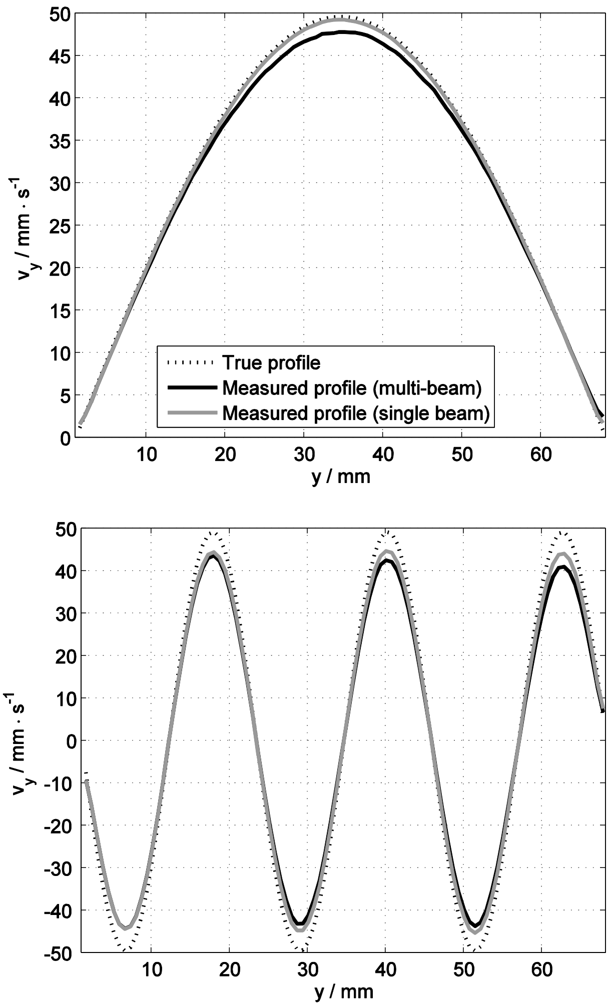

Figure 5: Averaged velocity profiles of an ultrasound simulation with 1×1 vortex cells (at the top) and 6×6 vortex cells (at the bottom). Given are the velocity profiles in multi-beam operation and in single beam operation as well as the specified true profile (legend at the bottom is the same as at the top).

The amount of the bias may vary for different measurement lines since the acoustic crosstalk may subtract or add a velocity bias depending on the velocity profiles of the neighboring ultrasound beams thereby partially compensating the bias induced by limited spatial resolution capabilities. For a better significance an error value for the entire measuring field is calculated in terms of the mean absolute error (MAE) normalized to v_0. This also enables to compare the results of different flow configurations among each other. The equation is given by:

$$\left\langle e_{v,MAE} \right\rangle = \frac{\frac{1}{N_x N_y} \sum_{n=1}^{N_x} \sum_{m=1}^{N_y} \left| v(m,n) - v_{true}(m,n) \right|}{v_o} \qquad (2)$$

where m, n are discrete variables for the two dimensions of the velocity field, v_{true} is the true velocity and v is the measured velocity.

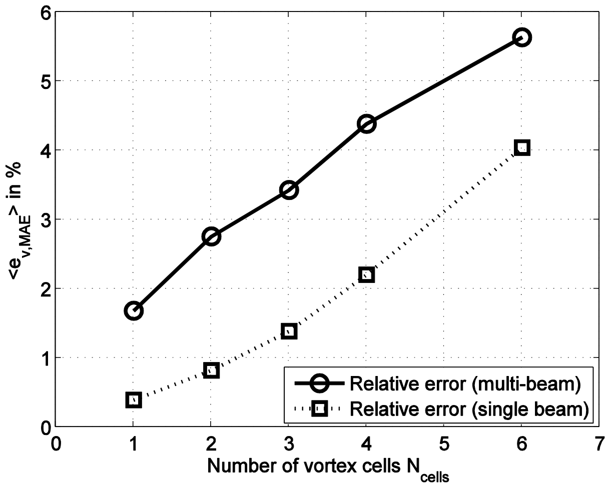

Figure 6: Normalized mean absolute error of different flow configurations.

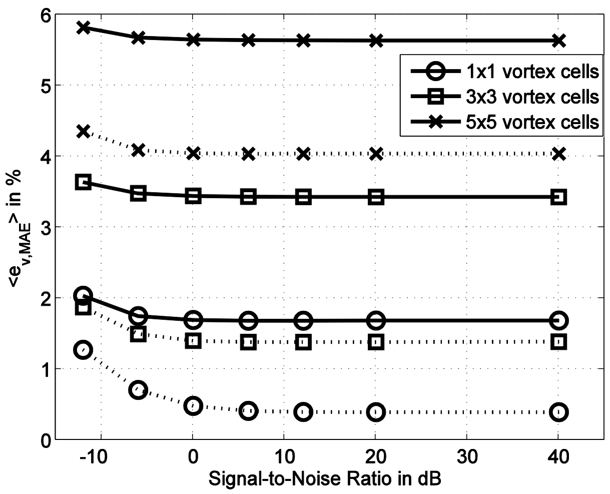

Figure 7: Error as a function of Signal-to-Noise ratio for different flow configurations. Solid lines for the error of multi-beam method and dashed lines for the single beam measurement.

In Fig. 6 the result of $e_{v,MAE}$ for different flow configuration is shown. It reveals that the bias induced by the multi-beam approach amounts around 1.3% to 2.2%.

For Fig. 7 white Gaussian noise is added to the echo signals for different flow configurations. The noise is identical for the multi-beam and single beam measurement to ensure the same measurement variance. Obviously from Fig. 7, noise has no significant influence on the crosstalk induced bias. For Signal-to-Noise Ratios (SNR) higher 0 dB the bias is almost constant.

5. Summary

A numerical model of a two-dimensional ultrasound Doppler flow mapping system is presented. It is mainly used for determining the systematic error (bias) of the measurement system, particularly the bias induced by the multi-beam approach of the system which measures with several beams simultaneously.

The calculation of the echo signals reflected from randomly distributed point scatterers moving in a defined flow field is performed by applying the system theory to linear acoustics. The simulation is carried out by the *FIELD II* toolbox for *MATLAB*. The theoretical Roberts flow is applied as flow.

Mainly, the number of vortex cells in the measuring field is varied for evaluating the bias. The received signals are calculated for the single and the multi-beam operation to compare their processed velocity data with each other. The comparison of the bias of both operation modes for different flow configuration reveals that the acoustic crosstalk increases the mean systematic error around 1.3% to 2.2% for the applied parameter set. In conclusion the error induced by the multi-beam approach is insignificant compared to the advantages (flow mapping with high frame rate and improved spatial resolution) of this measurement principle which denotes a progress for ultrasonic Doppler flow field measurements.

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