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Comparison of transition in concentric and eccentric Taylor-Couette flows

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1 Introduction

Some years ago it was hoped that an understanding of the transition to turbulence could be achieved by analyzing the data from a single time series via the method of delays (c.f. [3]). This hope was driven by the then impracticality and/or high cost of multi-point measurements. Now that instruments are being developed that allow the measurement of flow quantities at several spatial positions as a function of time, the emphasis is on extracting information about the *spatial* character and time-evolution of spatially extended systems.

In this contribution we report on our current investigation of the effect of rotational symmetry on the transition to turbulence in the Taylor-Couette system. Velocity measurements are obtained using the Ultrasound Velocimeter Profiler (UVP) [1, 2]. The data are analyzed using Singular Systems Analysis [3, 4, 5, 6] which allows an investigation of the space-time evolution of spatial patterns and their quantification as a function of Reynolds number.

2 Experiment

The experiments were performed at the Paul Scherrer Institute and the data was obtained using the ultrasound velocimeter profiler (UVP) (see [1, 2]). The axial component of the velocity field in the flow between two rotating cylinders was measured at 128 equidistant positions in the centre part of the system. The spatial resolution was 0.74 mm, and the time between two succeeding profiles was 135.3 msec. The maximum velocity that could be detected was 90 mm/s, and the velocity resolution was 0.7 mm/s.

The dimensions of our Taylor-Couette apparatus are: inner cylinder radius $a = 95$ mm, outer cylinder radius $b = 110$ mm, and cylinder length $L = 202$ mm. Thus the system geometry is characterized by the radius ratio $\eta = a/b = 0.8636$, and the aspect ratio

$\Gamma = L/(b-a) = 13.47$. Removeable end plates allowed the change from the concentric to the eccentric geometry. The eccentricity is defined by $\epsilon = c/(b-a)$, where c is the distance between the axes of the two cylinders. Results are given here for $\epsilon = 0$ and 0.28.

The Reynolds number Re is defined by $Re = a\Omega(b-a)/\nu$ where Ω is the rotational speed of the inner cylinder (the outer cylinder is at rest) and ν is the kinematic viscosity of the fluid. The fluid is a mixture of glycerol and water. Reynolds numbers are expressed as $Re^* = Re/Re_c$ where R_c is critical Reynolds number for the onset of Taylor-vortices.

For the concentric case axial velocity profiles were measured at only one angle, but for the eccentric case we measured the velocity profile at six angles spaced uniformly around the system. The angles reported here are given with respect to a coordinate system centred on the axis of the outer cylinder. The 0 degree position corresponds to the position of narrowest gap between the cylinders, and the angle increases in the direction of rotation of the inner cylinder. Measurements were carried out over the range $Re^* = (7.5, 30)$ for the concentric case and $Re^* = (8, 20)$ for the eccentric case.

3 Tools for Data Analysis

3.1 Singular Systems Analysis

The UVP technique enables us to measure the axial component of the velocity in the flow at 128 adjacent points in space during each sampling time. A time series of 1024 such velocity profiles are accumulated during each experimental run. The data is stored as a 1024×128 matrix V. Thus the j^{th} row of V is the velocity profile at time j, and the k^{th} column of V is the velocity time series at spatial position k. Most of the kinetic energy is contained in the motion of the fluid forming the Taylor-vortices. We are mainly interested in the fluctuation around this state. Therefore, we eliminate this dominating structure by subtracting the time averaged profile $\langle V \rangle_t$. That is, we subtract the mean value of each column from each entry of the column. In the following we will use the time-centred matrix X where $X = V - \langle V \rangle_t$.

We now carry out the singular system analysis of X [3, 4], which in the present case, since the columns label space, is the discretized version of the biorthogonal decomposition introduced by Aubry [5, 6]. The singular value decomposition of X is written

$$X = S\Sigma C^t \tag{1}$$

where $\Sigma = \text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_N)$ are the singular values. The columns of C are the *spatial* eigenvectors, and the columns of S are the *temporal* eigenvectors. We shall assume that the eigenfunctions have been ordered so that $\sigma_1 > \sigma_2 > \ldots > \sigma_N$. (See [3] for a guide to the computational details.)

The idea is that for a given Re the state of the system can be described by the spatial and temporal eigenvectors combined in pairs $\{(c_k, s_k)\}$, and their corresponding eigenvalues $\{\sigma_k^2\}$, the squares of the singular values. (Note that s_k is the time evolution of the spatial mode c_k.)

3.2 Representational entropy

Aubry *et al.* [5, 6] suggested that the signal spectrum be quantified by its *representational entropy* and investigated as a function of Reynolds number. The representational entropy is introduced by first defining the probabilities (normalized energies)

$$p_k = \sigma_k^2/E, \qquad E = \sum_k \sigma_k^2, \tag{2}$$

where E is the total energy of the signal. Then the entropy is given by

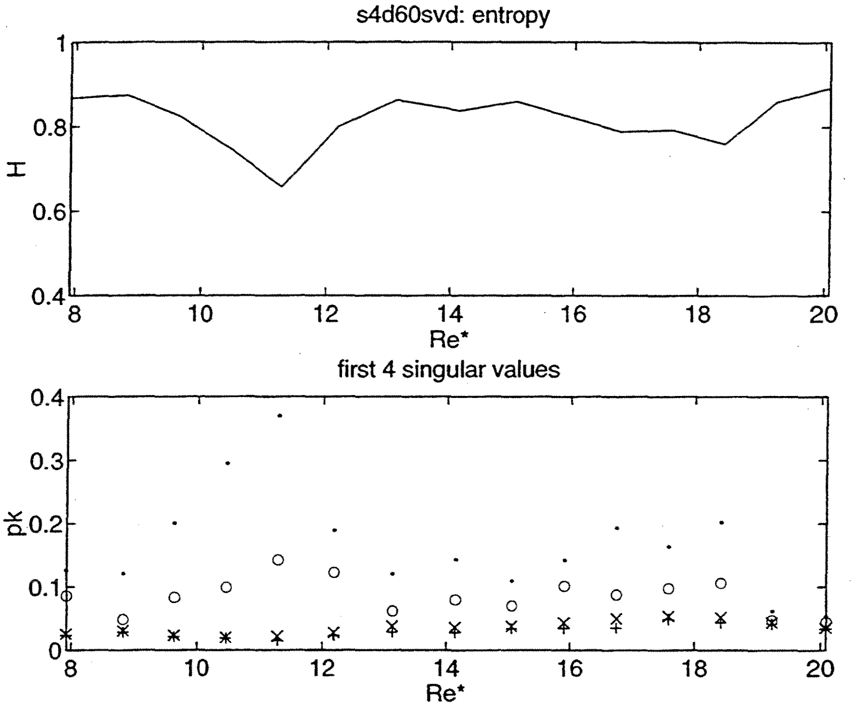
$$H(Re) = -\frac{1}{\log N} \sum_k p_k \log p_k \tag{3}$$

where N is the number of non-zero eigenvalues.

H is a measure of the complexity of the spectrum. If one single mode contains all the energy (i.e. $p_1 = 1$), it takes its minimum value $H = 0$. If the energy is uniformly distributed among all modes, then $p_k = 1/N$, and $H = 1$. During the transition to turbulence the entropy function is expected to increase as Re^* is increased since more and more modes are expected to participate in the complex motion of the fluid. By investigating H as a function of Re, one expects to observe features which mark qualitative changes in the flow structure (e.g., birth/death of a mode, mode competition, etc). (We note that we have also found that the number of eigenvectors containing 90% of the signal energy also provides a useful measure - especially in interpreting the onset of soft and/or hard turbulence.)

4 Results

A sample result of our investigation for $\epsilon = 0.28$ is shown in the figure where we plot the entropy (top), and the normalized energy p_k of the first four spatial-temporal eigenvectors as a function of Reynolds number. The velocities were measured at an angle of 60 degrees downstream from the position of narrowest gap. The dip in H at $Re^* \approx 11.5$ is suggestive of a change in the flow structure. We note however, that similar behaviour is seen at an angle of 120 degrees, but at $Re^* \approx 12.1$. All other angles indicate a rather complicated flow with no detectable change in structure over this range of Reynolds numbers. We hope to shed more light on these results soon.



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