Low frequency instability of the Karman vortex street: An experimental study of the secondary wake instability

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SUMMARY

The flow in the wake of a finite-length cylinder has been studied experimentally as it undergoes its secondary instability. This instability occurs at Reynolds numbers around 180-190. The energy content of four main frequency modes appearing in the wake was measured before and after transition. For three of the modes, the energy content remains largely unchanged by the transition process. However, after transition, a low frequency (turbulent) mode appears which exhibits energy four orders of magnitude greater after transition than before. By considering the downstream growth rate of this mode, evidence is presented which suggests that transition to turbulence may occur as a result of wake transition in the central downstream plane of the cylinder.

INTRODUCTION

The flow of a fluid around a circular cylinder has long served as one of the archetypal systems used for the study of wake formation in bluff body flow. The evolution of the wake up to Re$\sim$ 180 is now relatively well understood (Huerre and Monkewitz, 1990, Williamson, 1996a) but the secondary instability of the Karman vortex street, which for an infinitely long cylinder is known to occur at Re=188.5 (Henderson and Barkley, 1996) is still the subject of much discussion (Zhang et al, 1995, Williamson, 1996b, 1992). This secondary instability is of some importance because it is at this stage that turbulence, in the form of stochastic fluctuations of the velocity field, is first observed globally in the wake.

The presence of boundary conditions introduces complications that are now known to be of considerable importance. They influence the flow structure across the whole span of the cylinder, even in the case of large aspect ratios. In the work presented here experiments are carried-out on the wake of a finite-length cylinder placed in a boundary layer with the other end terminating in open flow (free end). Such a situation is more typical of engineering applications. Although, at first sight, these boundary conditions seem quite different with respect to other work carried out on cylinder flows, in practice it was found (Stocks et al, 1996) that they gave rise to a rather simple and stable wake structure that is similar in form to wake structures observed in other studies. This system is therefore ideal for the study of transition when oblique (rather than parallel) vortex shedding predominates.

Recent work by Williamson (1992, 1996b) suggests that natural transition to turbulence in cylinder wakes is a consequence of an amplification of vortex dislocations that are generated in the near wake of the cylinder. Experimentally it is known that at the secondary instability of the wake a new spatial mode is observed. This is often referred to as mode-A and is characterised by the appearance of streamwise vortices with a wavelength of approximately four cylinder diameters in the spanwise direction. It is at this secondary instability that additional randomly occurring dislocations seem to appear along the span of the cylin-

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der and, hence, turbulence is first observed. These dislocations grow as they propagate downstream. The results presented here seem to be consistent with this picture. It appears that the wake itself undergoes transition, resulting in the amplification of low frequency disturbances in the near wake.

THE WAKE STRUCTURE

Prior to transition, the wake has a rather simple structure that is now well understood. Directly behind the cylinder the flow is split into three separate spanwise frequency cells, a dominant central cell that corresponds to the main vortex shedding mode and two end cells which have lower shedding frequencies. Physically, this cellular structure arises through vortex dislocations at the cellular interfaces and is a direct consequence of the finite-end boundary conditions (Konig et al, 1992). These cellular modes can be thought of as the primary modes generated in the near wake. All other frequencies observed in the spectra (prior to transition to turbulence) are linear combinations of the frequencies of these modes The Strouhal frequency of the central cell was measured to be 163Hz, which corresponds to a Strouhal number ($S = fd/U$) of $S = 0.180$, this is to be compared with the predicted value for parallel shedding (Williamson, 1988) of 0.186. The discrepancy is due to the oblique nature of the shedding (this has been confirmed using measurements with two probes) which, using the formula (Williamson, 1988) $S_{obl} = S_{par}\cos(\theta)$, is calculated to have an angle $\theta = 14°$. At the interface between the cellular modes, a mode at their difference frequency (about 31Hz), is generated through nonlinear mixing. As this frequency is significantly smaller than the main vortex shedding frequency, it is found to be easily propagated downstream whilst the higher frequencies are rapidly filtered out (Williamson, 1992, Stocks et al, 1996). Consequently, it is this mode, generated by the vortex dislocations, that organises the far-wake region

After transition, a new low frequency mode appears which gives rise to a zero frequency peak in the power spectral densities. The magnitude of this peak is not small and surpasses the magnitudes of the main vortex shedding modes. Such a peak has recently been observed in another cylinder flow experiment (Williamson, 1996b), where large scale, low frequency fluctuations are observed to grow downstream of the cylinder. Evidence was presented that indicated that these low frequency fluctuations were a result of vortex dislocations which spontaneously appeared after the wake had undergone its secondary transition to spatial mode-A. However, the energy content of this peak was not studied in detail.

ENERGY CONTENT OF THE MODES

The evolution of the modes with downstream position, Figures 1, illustrate the variation of the average span-wise energy, $\langle E \rangle$, of each mode as a function of downstream position for $y/d = 0.0$. The energy of the centre cell mode, the end-cell modes, and the difference mode, Figs. 1(d,c,b) respectively, are seen to behave similarly both before (circles) and after transition (crosses). Hence these modes appear to be largely unaffected as the wake undergoes transition. This contrasts with the behaviour of the energy in the low frequency (turbulent) mode, shown in Fig. 1(a). After transition the energy in this mode increases by nearly four orders of magnitude. It is this increase in turbulent energy in the centre-plane that accounts for the greater part of the total energy in the wake.

The rapid growth (appearance) of the low frequency mode after transition results in the whole far-wake region becoming turbulent. Effectively, after transition, the turbulent fluctuations, already present in the near-wake region, become present in the rest of the wake. However, if the increase in turbulence were solely due to an increase in the fluctuational energy in the near wake, then one would not expect different growth rates before and after transition. This tends to suggest that the wake itself has undergone transition.

If one tries to interpret these results within the framework of Williamson's vortex dislocations picture, then one would speculate that it is the secondary instability of the vortex street, giving rise to transition to mode-A, that modifies the properties of the wake. This transition results in the central plane region becoming unstable to low frequency fluctuations. Hence, these fluctuations, which already exist in the near wake region prior to transition, are selectively amplified and give rise to large scale vortex dislocations. Whilst this is obviously a simplified view of what is a complicated spatiotemporal effect, we believe it may encompass the main features of the transition process

CONCLUSIONS

In conclusion we would propose that these results, taken with the mechanism proposed by Williamson (1992,1996b), suggest that the occurrence of turbulence is due to the selective amplification of low frequency fluctuations in the near wake. Our results also indicate that this amplification mechanism arises due to a transition of the wake itself. However, it appears that the wake properties are only strongly affected by this transition in the central downstream plane of the cylinder. We postulate that it is the transition to mode-A that actually causes the modification of the wake properties and hence to the appearance of the amplification mechanism. This would suggest that naturally occurring turbulence can only be observed after the formation of this mode.

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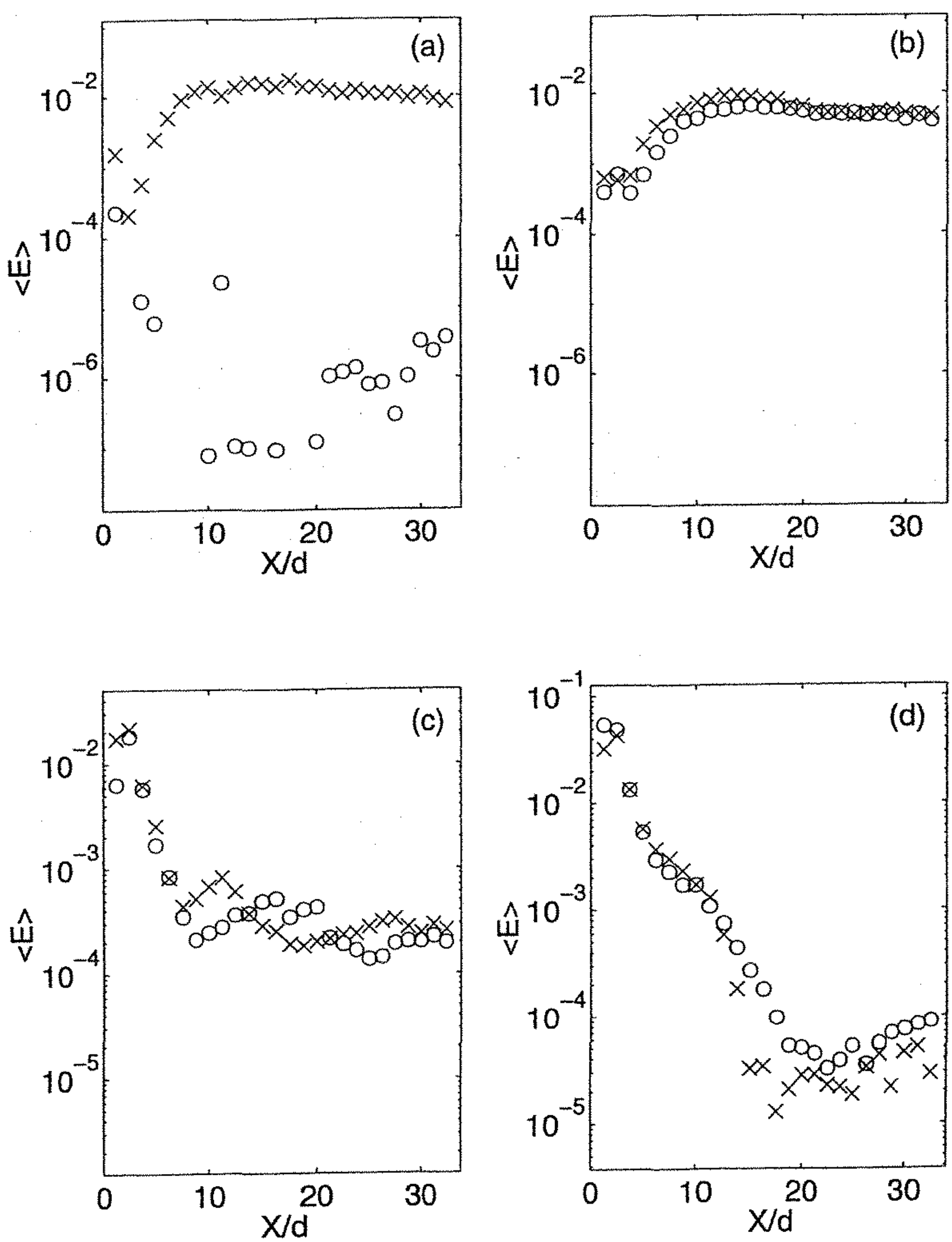

1. Average spanwise energy, $\langle E \rangle$, of modes as a function of downstream position in the centre-line plane $y/d = 0$. Four modes are shown: (a) the low frequency mode, (b) the difference frequency mode, (c) the end-cell mode and (d) the centre-cell mode. Circles represent the energy in a mode at Re=158. Crosses represent the energy in a mode at Re=189.