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Acoustic spectroscopy of Vorticity and Temperature fluctuations

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1 Introduction

This method was originally developped to obtain vorticity measurements. Indeed, in the study of hydrodynamics instabilities and turbulence, a flow is often described in terms of dynamics of vortical structures. Furthermore, in this context, interpetations are often given via mode (or Fourier components) interactions. However, vorticity measurements are scarce, and mostly indirect [1]. There was thus a strong motivation for the development of a direct, non-intrusive and spectral measurement of the vorticity.

The principle is that of classical spectroscopy: when a medium is exposed to an incoming wave the total scattered amplitude, far from the flow, is proportional to Fourier transform of the spatial disctribution of scatterers[1]. The nature of the wave choses the physical quantity being probed. Electromagnetic waves respond to density variations; sound waves couple with both with density variations and velocity gradients:
- in the case of density inhomogeneities, the speed of sound is locally modulated; the resulting scattering has a dipolar angular dependance.
- velocity variations are felt through inhomogeneities in the vorticity. Indeed, vortices scatter sound waves; the mechanism may be pictures as follows: a vortex is advected by the incoming wave and, being set into a non-stationary motion it radiates sound (in much the same manner as accelerated charges emit electromagnetic waves). The scattered wave has a quadrupolar structure.

Measurement of the forward scattered wave yields a measurement of the Fourier components of the vorticity, for an isothermal flow. When temperature fluctuations are present, they may be singled out by measurements of the backscatterd wave.

2 Theoretical background

The scattering a an ultrasonic wave by an inhomogeneous flows results from the non-linear coupling, through the Navier-Stokes equation, of the three hydrodynamics modes sound, entropy and vorticity [2, 3]. Although expressions for the scattered amplitude have been derived by a number of authors [4, 5, 6, 7]. compact and elegant formulations are given bay Lund and co-workers [8, 9]. For a plane incoming sound wave with frequency ν_0:

$$P_{\text{scat}}^{\text{vort}}(\vec{D},\nu) = P_0^{inc}\frac{e^{i\nu r/c_0}}{r}\;\frac{-\cos\theta\sin\theta}{1-\cos\theta}\;\frac{i\pi^2\nu}{c_0^2}\tilde{\Omega}_\perp(\vec{q},\nu-\nu_0)\;, \tag{1}$$

[1]It is assumed that thepopulation of scatterers is diluted so that Born's first approximation is justified.

$$P_{\mathrm{scat}}^{\mathrm{temp}}(\vec{D},\nu) = P_0^{inc}\frac{e^{i\nu r/c_0}}{r}\frac{\pi^2\nu^2}{c_0^2 T_0}\cos\theta\,\tilde{T}(\vec{q},\nu-\nu_0)\ , \tag{2}$$

at distance $\vec{D}$ from the flow, where $\vec{q} = \vec{k}_d - \vec{k}_i$ is the scattering wavevector, θ the scattering angle, c_0 the speed of sound at temperature T_0, $\Omega_\perp(\vec{r},t)$ the component of the vorticity field perpendicular to the scattering plane and $T(\vec{r},t)$ the temperature field (we write $\tilde{F}$ the Fourier transform of a function F).

3 Experimental implementations

Generalities

The scattering wavenumber choses the scale at which the flow is probed, $q = 4\pi\nu_0\sin(\frac{\theta}{2})/c$. In experiments, this is conveniently achieved by tuning the frequency of the incoming sound wave. In laboratory flows the scales of motion are in the range 10cm to 100μm; they are resolved using ustrasonic transducers which emit in the band 100kHz to 50MHz for flows in liquids (commercially available) or in the band 5kHz to 400kHz for flows in gases (custom made [10]). The transducers must be as large as possible to define a scattering wanuber with precision[2]; typical values are $\Delta q/q \sim 2\%$.

Validation

We have extended the early work of the 60s russian acoustic school [11, 12] to develop sound scattering as a spectroscopic tool for flow investigation. First equations (1) and (2) have been validated in simple flows where scatterers are periodically spaced. For vorticity such is the case of the von Kármán vortex street; for thermal perturbations we have used the thermal wake above a heated wire. In each case [13, 14, 15], we have established the spectral nature of the measurement and shown that this scattering technique allows original contributions to the study of spatio-temporal instabilities.

Applications to turbulence

We have studied a heated jet [16], in a case where temperature may be regarded as a passive scalar. The scattering technique gives a direct measurement of the spectrum, in space, of the temperature fluctuations; we have observed a spectral density $T^2(q) \propto q^{-5/3}$ in agreement with the Corrsin-Obhukov theory. We have also shown the intermittency increases at small scale in agreement with the formation of scalar fronts.

Vorticity measurements in a jet [17] have shown that the enstrophy spectrum has a $q^{1/3}$ inertial behavior, in agreement with Kolmogorov's mean field theory. Regarding intermittency, we have shown [18, 19] that the Fourier components of vorticity have a trivial behavior at all scales, save for a band around the Taylor microscale where intense filaments are formed.

Current research is oriented towards simultaneous dynamical analysis at several wavenumbers, to probe directly the triad interactions that underly the turbument cascade [20, 21].

[2]But the measurement volume must remain in the far field.

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