Velocity Profile of Non-magnetic Fluid and Magnetic Fluid Sloshing

Shin-ichi Yoshida, Kenji Tomita, Andrea Benvenuti, Tatsuo Sawada
and Masaaki Motozawa
Department of Mechanical Engineering, Keio University
3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan

The dynamic behavior of a two-layered fluid in a laterally oscillated rectangular container under a vertical non-uniform magnetic field was studied experimentally and theoretically. The two-layered fluid was formed of water based magnetic fluid and silicone oil. In the experimental approach, the dynamic behavior was studied by measuring the velocity profile of the two-layered fluid using an ultrasound velocity profile measuring technique while varying the vertical magnetic field. In the theoretical approach, analytical results are obtained from nonlinear theory using perturbation method. The effect of the magnetic field on the sloshing of the two-layered fluid was considered.

Keywords: Magnetic Fluid, Sloshing, Ultrasonic Velocity Profile Method, Rectangular Container, Non-uniform Magnetic Field

1 INTRODUCTION

A magnetic fluid is a stable colloidal solution of many surfactant-coated ferromagnetic particles in a liquid carrier. When magnetic fluids are used as a work fluid of a fluid mechanical system, it is necessary to know the flow characteristics. However, we cannot measure internal velocity profiles using optical methods because magnetic fluid is opaque.

The Ultrasound Velocity Profile (UVP) method [1-2] is a method of measuring a velocity profile on a line with respect to the velocity component along the ultrasonic beam. This technique has a typical advantage in comparison with ordinary methods. It can be applied to an opaque fluid like a magnetic fluid [3-4].

Several studies have been performed to investigate liquid sloshing [5]. Sawada et al. [6] analyzed sloshing using a magnetic fluid in a rectangular container subject to non-uniform magnetic fields and measured inner velocity profiles using the UVP method. Although these theoretical and experimental studies have been carried out to better understand sloshing, the mechanisms involved in two-layered fluid sloshing with a magnetic fluid remain unclear and inner velocity profiles of such a system have not been measured as yet.

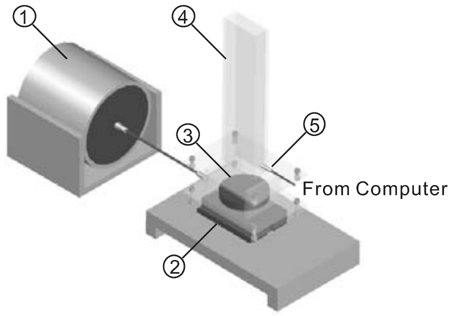
In the present paper, we experimentally measured the displacement of the free surface and the inner velocity profiles of a two-layered fluid sloshing under a non-uniform magnetic field. The test fluid included layers of a water-based magnetic fluid and a silicone oil. The influence of magnetic field on resonant frequency and inner velocity profiles are presented.

2 TEST FLUIDS

A water based magnetic fluid (W-40) and a silicone oil (KF-56) are used. W-40 has a 40 % weight concentration of fine magnetic particles and is produced by Taiho Industries Co., Ltd., Japan. Its sound velocity[6], viscosity and density are c = 1440 m/s, μ = 25 mPa·s and $\rho = 1400$ kg/m^3 at 25 °C. A porous SiO$_2$ powder (MSF-10M, Liquidgas, Co., Ltd.) having grains with an average diameter of about 0.9 mμ is added to provide reflectors for the ultrasonic waves. KF-56 is produced by Shin-Etsu Chemical, Co., Ltd., Japan. Its sound velocity, viscosity and density are c = 1250 m/s, μ = 15 mPa·s and $\rho = 1400$ kg/m^3 at 25 °C. A nylon powder produced by Ono Science Co. Ltd. is added in order to provide reflectors for the ultrasonic waves for KF-56. The average diameter of grains is about 4 μm.

3 EXPERIMENT

Figure 2 shows the experimental apparatus and Figure 3 shows the test rectangular container. The rectangular container measures 90 mm × 30 mm × 380 mm and is made of acrylic resin. The oscillator has a frequency range from 0.8 Hz to 3.5 Hz. The amplitude of oscillation is 1.0 mm for all experiments. Three permanent magnets are used. Their averaged magnetic flux densities are as follows: 16.9 mT, 24.3 mT and 34.6 mT. An ultrasonic transducer is fixed on the side wall of the container in order to measure



① Vibration Exciter ③ Magnet
② Roller Table ④ Rectangular Container
⑤ UVP Probe

Figure 1: Experimental apparatus

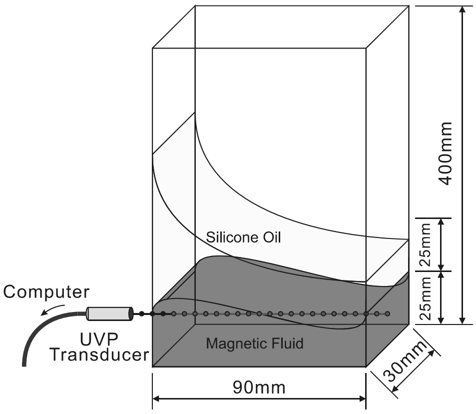


Figure 2: Test container

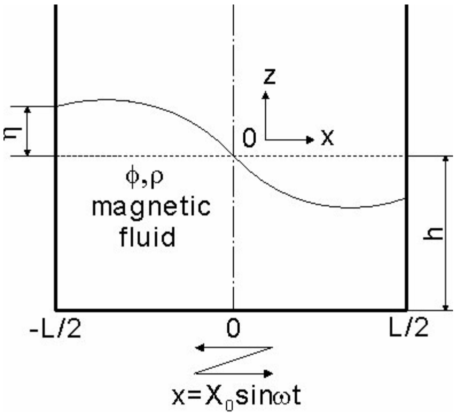


Figure 3:analytical model

the horizontal velocity profile. The ultrasonic transducer has an active diameter of 5 mm and the measuring volume is a thin-disc shape element $\phi 5$ mm × 0.71 mm. The basic frequency is 4 MHz. LabVIEW is used in order to transmit and to receive the ultrasonic pulse wave.

4 THEORY

The flow behavior of sloshing is essentially nonlinear. In particular, forced oscillation clearly shows nonlinear characteristics in its resonance curves. Thus, a nonlinear approach is necessary to investigate velocity fields and high frequency responses of the free surface. Here we deal with an analytical solution for sloshing subject to non-uniform magnetic field using a perturbation method. An analytical model is shown in Figure 4. Here *x*-axis is horizontal while *z*-axis is measured vertically from the mean position of the free surface. *L* is the length of the container and η is the free surface elevation. X_0 and ω are the amplitude and forced angular frequency, respectively.

With the assumptions of irrotational flow and an incompressible fluid, a velocity potential ϕ exists which should satisfy the continuity equation:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2}=0 \tag{1}$$

The unsteady Bernoulli equation is given by

$$\begin{aligned}&\frac{\partial\phi}{\partial t}+\frac{1}{2}\left\{\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2\right\}+\frac{p}{\rho}+gz-\frac{1}{2}\mu_0\chi_m H^2\\&\quad=X_0\omega^2 x\sin\omega t\end{aligned} \tag{2}$$

where *p*, *g*, μ_0, *H* and χ_m are pressure, gravitational acceleration, magnetic permeability of vacuum, magnetic field and susceptibility of a magnetic fluid respectively. The magnetic field *H* is approximated by

$$H(z)=H_0e^{-\alpha(z+h)} \tag{3}$$

where H_0 is the magnetic field intensity at the bottom of the container ($z = -h$) and α is a constant which is determined by measurement of the magnetic field. The boundary conditions on the bottom and side walls, and the kinematical and dynamical free surface conditions are given by following equations:

$$\left(\frac{\partial\phi}{\partial x}\right)_{x=\pm L/2}=0 \tag{4}$$

$$\left(\frac{\partial\phi}{\partial z}\right)_{z=-h}=0 \tag{5}$$

$$\left(\frac{\partial\eta}{\partial t}+\frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x}\right)_{z=\eta}=\left(\frac{\partial\phi}{\partial z}\right)_{z=\eta} \tag{6}$$

$$\begin{aligned}&\left[\frac{\partial^2\phi}{\partial t^2}+2\left(\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x\partial t}+\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial z\partial t}\right)+\left\{\left(\frac{\partial\phi}{\partial x}\right)^2\frac{\partial^2\phi}{\partial x^2}+\left(\frac{\partial\phi}{\partial z}\right)^2\frac{\partial^2\phi}{\partial z^2}\right\}\right.\\&\qquad+2\frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial z}\frac{\partial^2\phi}{\partial x\partial z}+g_m\frac{\partial\phi}{\partial z}-X_0\omega^3x\cos\omega t\\&\qquad\left.-X_0\omega^3\frac{\partial\phi}{\partial x}\sin\omega t\right]_{z=\eta}=0\end{aligned} \tag{7}$$

where g_m is an effective gravity due to the magnetic force and is given by

$$g_m=g+\frac{\alpha\mu_0\chi_m H_0{}^2e^{-sah}}{\rho} \tag{8}$$

Since Eqs. (6) and (7) for the free surface conditions are nonlinear, they are linearlized by developing ϕ and η into power series of small parameter $\varepsilon = (X_0/L)^{1/3}$ up to the third order perturbation. These expressions are written in the following dimensionless forms:

$$\phi^*=\frac{\phi}{L^2\omega_1}=\varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+O(\varepsilon^4) \tag{9}$$

$$\eta^*=\frac{\eta}{L}=\varepsilon\eta_1+\varepsilon^2\eta_2+\varepsilon^3\eta_3+O(\varepsilon^4) \tag{10}$$

Where ω_1 is the first resonant angular frequency obtained by linearlized theory and where the function *O* represents other perturbation orders. The first resonant frequency ω_1 is given by

$$\omega_1=\sqrt{\frac{\pi g_m}{L}\tanh\frac{\pi h}{L}} \tag{11}$$

Space and time variables are also defined in the following dimensionless forms:

$$x^* = \frac{x}{L}, \quad z^* = \frac{z}{L}, \quad t^* = \omega t \tag{12}$$

Since we are interested in the behavior at the vicinity of the resonant point, we write angular frequency ω as follows:

$$\frac{\omega}{\omega_1} = 1 + \varepsilon\omega_1{}^* + \varepsilon^2\omega_2{}^* + O(\varepsilon^3) \tag{13}$$

Using the mathematical preparations, Eqs. (6) and (7) are linearized and we obtain the following nonlinear solutions of η_1, η_2 and η_3:

$$\eta_1 = a \sin \pi x^* \sin t^* \tag{14}$$

$$\eta_2 = a^2 A_{22} \cos 2\pi x^* \cos 2t^* \tag{15}$$

$$\begin{aligned}\eta_3 = {} & a^3 A_{11} \sin \pi x^* \sin t^* - a^3 A_{31} \sin 3\pi x^* \sin t^* \\ & + a^3 A_{13} \sin \pi x^* \sin 3t^* + a^3 A_{33} \sin 3\pi x^* \sin 3t^* \\ & + \frac{2T_H}{\pi} \sin \pi x^* \sin t^* \\ & + \frac{4T_H}{\pi} \sum_{n=2}^{\infty} \frac{(-1)^{n=1} \sin(2n-1)\pi x^*}{(2n-1)^2 \{1-(\omega_1/\omega_{2n+1})\}^2} \sin t^*\end{aligned} \tag{16}$$

where ε satisfies the following equation:

$$R\varepsilon^3 a^3 - \left(\frac{\omega}{\omega_1} - 1\right)\varepsilon a - \frac{2T_H X_0}{\pi L} = 0 \tag{17}$$

and T_H, R and A_{ij} are given by

$$T_H = \tanh\left(\frac{\pi h}{L}\right)$$

$$R = \frac{\pi^2}{64}\left(9T_H{}^{-4} - 14T_H{}^{-2} - 3\right)$$

$$A_{11} = \frac{\pi^2}{64}\left(3T_H{}^{-4} - 12T_H{}^{-2} - 1\right)$$

$$A_{12} = \frac{\pi^2}{256}\left(9T_H{}^{-4} + 54T_H{}^{-2} - 15\right)$$

$$A_{22} = \frac{\pi}{8}\left(3T_H{}^{-3} - 12T_H{}^{-1}\right)$$

$$A_{31} = \frac{\pi^2}{256}\left(27T_H{}^{-4} + 51T_H{}^{-2} - 87 - 15T_H{}^{2}\right)$$

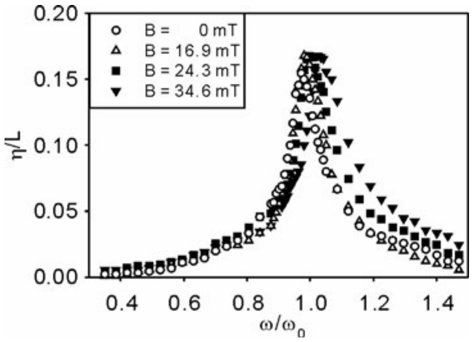


Figure 4: Frequency response of surface elevation of the magnetic fluid

$$A_{31} = \frac{\pi^2}{256}\left(81T_H{}^{-6} - 27T_H{}^{-4} - 27T_H{}^{-2} - 9\right)$$

5 RESULTS AND DISCUSSIONS

5.1 Magnetic Fluid Sloshing

The frequency response of the free surface of magnetic fluid sloshing is shown in Figure 4. The fluid height is 50 mm. Here *B* is the surface average magnetic flux density, η is the maximum free surface elevation at the side wall and ω_0 is the resonant angular frequency obtained experimentally for *B* = 0 mT. This frequency is *f* = 2.775 Hz. As the forcing frequency increases, the surface elevation also increases until the free surface is intensively shaken near the resonant frequency. After the resonant frequency, the surface shaking is suppressed. As the magnetic field intensity increases, the first resonant frequency shifts to a higher frequency. For *B* = 34.6 mT, the first resonant frequency is 2.925 Hz.

In Figure 6 experimental results are compared with the linear and nonlinear theoretical results for *B* = 16.9 mT. In the low frequency region, the nonlinear solution is larger than the experimental values because calculated amplitude does not become zero with decrease of the frequency. Experimental and nonlinear theoretical results have a good agreement, especially in the high frequency region. UVP measurements were carried out at the first resonant frequency. Figure 6 shows the magnetic field dependent velocity profiles. Here the start phase is the minimum surface elevation time and *z*

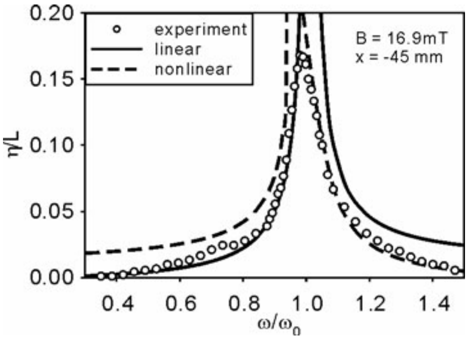


Figure 5: Comparison of theoretical and experimental results for frequency response of surface elevation

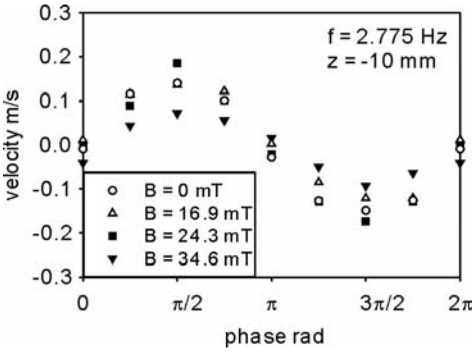


Figure 6: Time dependent velocity profiles of the magnetic fluid at the middle point

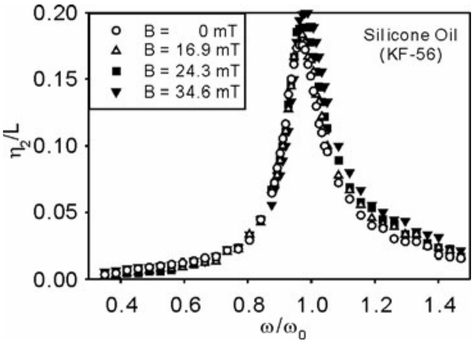


Figure 7: Frequency response of surface elevation for two-layered fluid sloshing

corresponds to the position of the UVP transducer from the static free surface of a magnetic fluid.

5.2 Two-layered Fluid Sloshing

The frequency response of the free surface of the two-layered fluid sloshing is shown in Figure 7. The fluid height of each fluid is 25 mm. Here η_2 is the maximum free surface elevation of a silicon oil at the side wall. As the forcing frequency increases, the surface elevation also increases until the free surface is intensively shaken near the resonant frequency. This tendency is similar to the result for single magnetic fluid sloshing. After the resonant frequency, the surface shaking is reduced. Similar to the above, the first resonant frequency shifts to a higher frequency as the magnetic field intensity is increased. For B = 34.6 mT, the first resonant frequency is 2.825 Hz. In this case, the rate of change of the resonant frequency and increasing rate of the surface elevation are smaller than those for single magnetic fluid sloshing because the magnetic force is relatively small.

Figure 8 shows the magnetic field dependent velocity profiles of two-layered fluid sloshing. Here z_2 corresponds to the position of the UVP probe from the interface between the two layers. Comparing Figures 6 and 8, there do not appear to be significant differences. The rate of change of the velocity in the two-layered sloshing is slightly smaller. The expected reduction of the inner velocities according to the magnetic field does not occur. In the present experiment, we have used a constant fluid volume ratio, i.e. 1:1. In the future, it will be useful to perform more detailed experiments while changing the arrangement/combination of the two layers.

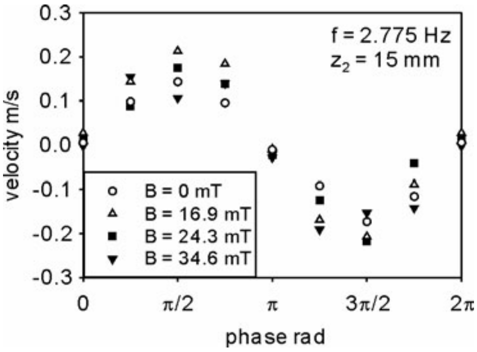


Figure 8: Time dependent velocity profiles for two-layered fluid sloshing

SUMMARY

Lateral sloshing of a magnetic fluid and a two-layered fluid in a rectangular container in a vertically-applied non-uniform magnetic fields has been investigated. In magnetic fluid sloshing, frequency response of the free surface and the inner velocity profile can be controlled using a magnetic field. In two-layered fluid sloshing, frequency response of the free surface and the inner velocity profile are generally similar to those in a single magnetic fluid. However, there appears to be the possibility for reducing the disturbance of the upper fluid by using magnetic fluid for the lower layer and applying a magnetic field.

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