Appearance of 3D structures on quasi-2D convection rolls in a Rayleigh-Bénard convection imposed by horizontal magnetic field

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We investigate the development of 3D structures on quasi-2D convection rolls of Rayleigh-Bénard convection in a liquid metal layer with an imposed horizontal magnetic field using UVP. Measurement lines arranged parallel to the magnetic field effectively represent 3D structures on the convection rolls. As the fundamental 3D structure, existence of horizontal circulations parallel to the axis of convection rolls was elucidated. Scale estimation suggests that the circulation is caused by Ekman pumping due to increase of rotation speed of the rolls and thinning of velocity boundary layer on the side wall perpendicular to the magnetic field as typical MHD effects. On the four-roll state that is observed in a parameter region where the Lorentz force against the buoyancy is relatively weak, couple of thinner vortices wrapping around the primary convection rolls and advection along the horizontal circulation was observed. Oscillating motion of the main rolls is modulated by these advections. Parameter region for the appearance of this 3D structure is very close to that for the flow reversals and it may have importance to determine the appearance of the flow reversals.

Keywords: Ultrasonic flow visualization, MHD, Rayleigh-Bénard convection, Flow instability

1. INTRODUCTION

The application of a horizontal magnetic field to the liquid metal Rayleigh-Bénard convection stabilizes complex three-dimensional motions into quasi-two dimensional convection rolls because of the effect of Lorenz force. This MHD-Rayleigh-Bénard convection is governed by three non-dimension parameters, Rayleigh number ($Ra = \beta g \Delta T L^3/\kappa\nu$) indicating the balance between buoyancy and viscous force, Chandrasekhar number ($Q = \sigma B^2 L^2/\rho\nu$) indicating the balance between Lorentz and viscous force, and Prandtl number ($Pr=\kappa/\nu$, 2.3×10^{-2} in GaInSn), where $\beta, \nu, \kappa, \sigma, \rho$ are respectively, bulk modulus, kinematic viscosity, thermal diffusivity, electric conductivity, and density of a test liquid metal, and g, ΔT, L, and B are respectively, gravity acceleration, vertical temperature difference, height of the fluid layer, and intensity of the magnetic field.

In a recent paper [1], we reported the regime diagram of the convection patterns including the rolls-number dependence for Rayleigh number and Chandrasekhar number in a square vessel of aspect ratio five. In cases of moderate intensity of the magnetic field, various flow patterns appear depending on a balance between Lorenz force due to the applied magnetic field and the buoyancy. With decreasing intensity of the magnetic field, the quasi-2D convection rolls arrange parallel to the magnetic field assumes considerable 3D structures. Our group represented regime transitions including characteristics phenomena termed ‘flow reversals’ with Ra and Q [2]. This phenomenon might be caused by oscillatory instability and skewed varicose instability on quasi-two dimensional rolls. There is, however, still very small knowledge about 3D development of the flow structures.

In this report, we provide detailed representation of the 3D development by investigating transitions on spatiotemporal maps of flow velocity component parallel to the magnetic field reflecting 3D modifications of structures by ultrasonic velocity profiler (UVP), which is a good tool to visualize flow patterns in opaque fluids like liquid metals. Doing that in addition to velocity profile measurements perpendicular to the magnetic field by UVP, the development of 3D structures on quasi-2D convection rolls of Rayleigh-Bénard convection can be observed.

2. APPARATUS AND METHOD

Figure 1 shows the setting of the experimental container that has the same geometry as in the previous study [1]. The test section has dimensions of 200 mm × 200 mm × 40 mm leading to an aspect ratio of 5. The top and bottom plates are made of copper. The temperature of each plate was kept constant by circulating water via a thermostatic bath. The container was filled with the gallium-indium-tin alloy (GalnSn) of which melting point is -19 degree of Celsius, and thus it is in liquid state at room-temperature, is commercial liquid metal alloy composed of 67% of gallium, 20.5% of indium and 12.5% of tin. The fluid temperature was constant during each parameters. A DOP2000 velocimeter (Signal Processing SA) was used to measure the fluid velocities. The device is connected to 8 MHz transducers which are set in holes in the side wall made of polyvinyl chloride and are in direct contact with the test fluid. The velocity components perpendicular to the magnetic field were measured by two transducers (ch1, ch2), while the velocity components parallel to magnetic field were measured by six transducers (ch3-8). The UVP measures the projected flow velocity along each line, and thus the present arrangement of the transducers provides two velocity components, parallel and perpendicular to the magnetic field. The temperature fluctuations were measured by thermocouples at three points (Fig. 1). The head of each thermocouple was located 3 mm below the top surface of fluid layer. A magnetic generator consisting of a pair of rectangular shaped coils creates quasi-uniform horizontal magnetic field with an intensity of 300 mT as the maximum.

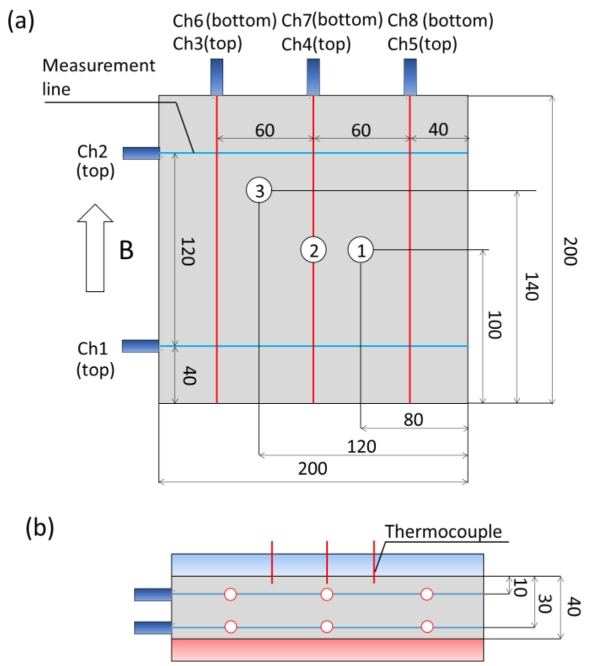

Figure 1: Geometry of the experimental container; (a) top view of the vessel and UVP measurement lines, and (b) side view and locations of each sensor

3. RESULTS AND DISCUSSION

3.1 Flow structure parallel and perpendicular to magnetic field in 5 roll configuration

Spatio-temporal velocity distributions perpendicular and parallel to the magnetic field were measured at $Ra = 1.26 \times 10^4$ and $Q = 3.38 \times 10^3$. Figure 2 shows the velocity distribution along the four measurement lines (Ch1, Ch6-8) for 2000 s. The distribution at Ch1 confirms that five-roll structure arranged parallel to the magnetic field exists stably. In the direction parallel to the magnetic field, the velocity distribution is separated into two regions having opposite flow directions at the middle of measurement distance in Ch6-8. In comparison with the velocity field perpendicular to the magnetic field, the magnitude of velocity component parallel to magnetic field is smaller, but not negligible, around 1/5. Though the flow achieves a highly quasi-two dimensional structure shown by Ch1 accompanied by a weak structure in the direction parallel to the magnetic field.

The flow direction shown in Fig. 2 is opposite between Ch6, Ch8 and Ch7. Checking the arrangement of transducers (Fig. 1) against arrangement of rolls (Fig. 2 Ch1), Ch6 and Ch8 are placed near the edge of a convection roll, while Ch7 is placed at the center of a convection roll. These results indicate that two horizontal circulations exist inside the roll structure. Figure 3 shows the illustration of this circulation expected from the velocity profile measurements. This circulation is directed from the wall to the center of the vessel at the center of rolls and the opposite direction at the outside of roll structure. This circulation may be caused by the formation of Ekman pumping inside the Hartman layers: In MHD flows, existence of electrically-insulated walls perpendicular to the magnetic field makes velocity boundary layers thinner because of induced electric current in the fluid layer (Hartman layer) [3]: Ekman pumping induces fluid motion perpendicular to walls by the conversion of divergence at a boundary layer in a rotating flow field. The order of flows due to Ekman pumping is estimated to compare with the horizontal circulation observed. Induced velocity component normal to the wall, w_e driven by Ekman pumping is estimated by thickness of boundary layer δ and normal vorticity component against the wall at the surface ω_z defined as, $w_e = 1/2 * \delta\omega_z$. Assuming that the roll structure takes rigid body rotation according to the isothermal core of usual Rayleigh-Bénard convections and roll size ($D \sim 40$ mm) is diameter of the rigid body, characteristic velocity U is defined as rotation speed of the roll. This is defined as $\delta_H = (\rho\nu/\sigma B^2)^{1/2}$ which depends only on the

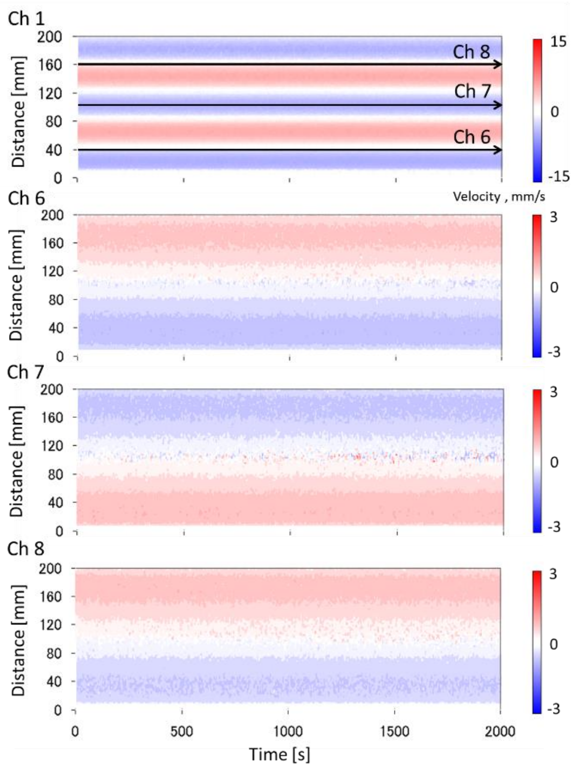

Figure 2: Spatio-temporal velocity distribution measured by UVP at each transducer; Ch1 perpendicular to the magnetic field; Ch6-8 parallel to the magnetic field ($Ra = 1.26 \times 10^4$ and $Q = 3.38 \times 10^3$)

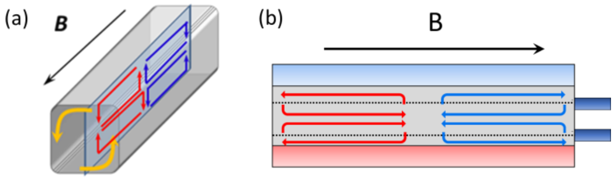

Figure 3: Schematic views of the horizontal circulation with five rolls; (a) the flow direction in the roll structure with quasi-two dimensional convection, (b) the overall view of horizontal circulation inner roll structure at side view

fluid property and the strength of the magnetic field [4]. According to this formula, thickness of the Hartman layer is estimated as $\delta_H = 6.89\times10^{-1}$ mm. The vorticity is $\omega_z = O(U/D)$, so the velocity due to Ekman pumping is estimated as $w_e = O(10^{-1})$. The magnitude of time-averaged velocity parallel to the magnetic field is 0.49 mm/s and assents to the estimated order of Ekman pumping.

3.2 Appearance of 3D structure in four roll

Velocity distributions displayed in Fig. 4 show the results of the measurement at $Ra = 1.2 \times 10^4$ and $Q = 5.12 \times 10^2$, where a four-roll structure is emerged. The result at Ch2 perpendicular to the magnetic field confirms formation of the four rolls with small oscillations. In the results of Ch6-8 parallel to the magnetic field, characteristic small scale flow structures appear, while the roll structures are maintained (Fig. 4 Ch6-8). These small structures are advected from the center to the side-walls of the vessel and are superimposed with small fluctuations. These small fluctuations might be caused in connection with the oscillation of the roll structure. Comparing the signals of the transducers placed at the bottom and the top of fluid layer, it seems that these flow structures span around the entire convection roll circumference.

To characterize the roll and small scale structures, Fig. 5 shows the results of two-dimensional Fourier transform calculated from spatiotemporal velocity distributions at Ch2 and Ch8, where the vertical axis represents wavenumber k, the horizontal axis represents frequency f. Here the sampling time is 2.4 s and the spatial resolution is 1.37 mm. It is observed that two peaks appear on the spectra of Ch2 and Ch8 at f = 0.003 and 0.03 Hz. The former frequency indicates the cycle of emitted vortex generations. The period corresponding to this frequency is about 330 s. This agrees with the period observed on the spatiotemporal velocity distributions.

The latter frequency indicates the frequency of the short time oscillation on the roll structures, because the corresponding period, about 33 s, agrees with the circulation time with considering elliptic shape for the rolls, 31.4 s (e.g. [3]). For the estimation we assumed that length of circulation on a roll is ~ $2\pi \times 10$ mm, where 10 mm equals the displacement of the measurement line from the center of the rolls, and the typical flow velocity is around 4 mm/s. This oscillation of roll structure affects the advective vortices, because the vortices move along the horizontal circulation created by the primary rolls. In comparison with the spatiotemporal map shown in Fig. 4, the former frequency component corresponds to the cycle of the vortex behaviour, generation at the center of vessels, advection along the horizontal circulation, and disappearance at the side wall. The corresponding wave number k on the spectrum of Ch2 is $k = 0.019$ mm^{-1}, this value shows the size of roll structures. It is assumed that the oscillation of the former frequency at each roll is influenced by the appearance of these vortices.

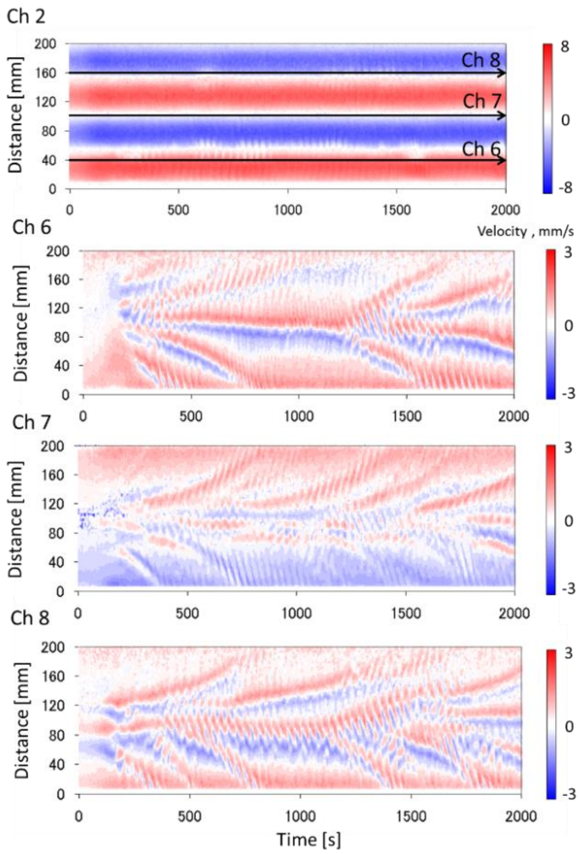

Figure 4: Spatio-temporal velocity distributions in the four roll structure ($Ra = 1.2 \times 10^4$ and $Q = 5.12 \times 10^2$), Ch2 perpendicular, Ch6-8 parallel to the magnetic field

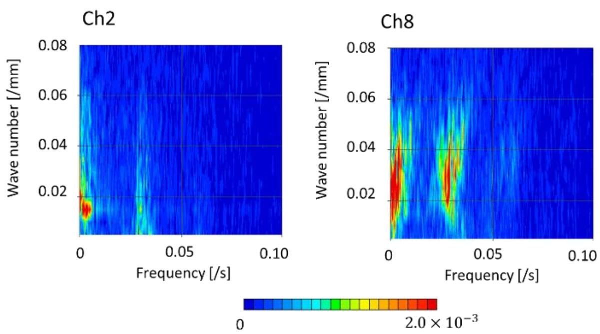

Figure 5: Two-dimensional Fourier transform of the spatiotemporal velocity distribution of Fig. 4 at Ch2 and Ch8, where the horizontal axis represents frequency f the vertical axis represents wavenumber k.

We may be able to find the corresponding flow structure in results of numerical simulations previously done by our group to reproduce the experimental results [5]. Figure 6 shows examples of the simulations, (a) Ra = 1.0 × 10^4 and Q = 1.0 × 10^3, (b) Ra = 3.0 × 10^4 and Q = 1.0 × 10^3. The setting parameters do not agree completely with the experiments, but each result qualitatively represents flow structures expected from Fig. 2 and Fig. 4, respectively; Fig. 6(a) shows five-roll condition, where the rolls stably exist; Fig. 6(b) shows, primary four rolls and secondary fine vortices around primary ones. It is assumed that transducers mounted parallel to the magnetic field detected these thinner vortices.

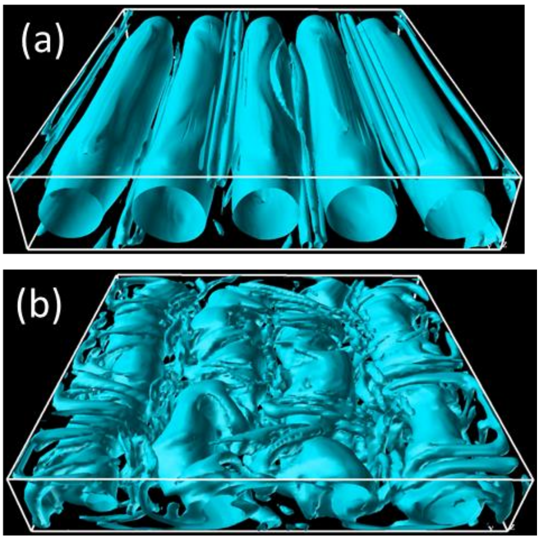

Figure 6: Appearance of secondary vortices around primary convection rolls ibserved in numerical simulation; (a) Ra = 1.0 × 10^4 and Q = 1.0 × 10^3, (b) Ra = 3.0 × 10^4 and Q = 1.0 × 10^3 [5]

Appearance of these thinner vortices suggests that secondary instability occurs. The mechanism behind this instability is explained as follows. In the laminar state, the flow in the convection vessel is composed of the main convection rolls, aligned along the direction of magnetic field. In addition to this, there are much smaller rolls located at the vessel corners and between neighbouring rolls (see figure 6(a)). The axis of these smaller vortices is parallel to the primary convection rolls. If the rotational speed of the primary convection rolls exceeds a certain threshold, the smaller corner vortices are entrained and wind around the primary rolls. The consequence is a counter rotating vortex pair that spans around the perimeter of the primary convection rolls and whose visual appearance looks similar to Taylor-Görtler vortices [6].

Previous work indicated that the regimes taking different number of rolls can be organized by Ra/Q on the Q - Ra plane [1]. The fraction of Ra/Q is a measure for the ratio of the buoyancy to the Lorenz force. This flow structure with thinner vortices appears at $Ra/Q \sim 10$ that corresponds to the region of Ra/Q to observe the flow reversals [1]. Appearance of these 3D structures may be connected to the flow reversals, especially restriction of the range of Ra/Q and hysteresis.

4. CONCLUSION

Appearance of 3D structure on quasi-2D convection rolls with imposed horizontal magnetic field was investigated by the measurement of velocity profiles parallel to the direction of magnetic field. As the fundamental 3D structures, horizontal circulation along the roll axis aligned parallel to the magnetic field, was observed through the experiments. The magnitude of this circulation is about 1/5 of speed of rotation of the rolls. Assuming that the roll structure takes rigid body rotation like a circular cylinder and applying some parameters in the formula of Ekman pumping, estimated velocity of wall-normal flows is comparable to the measurement results of circulation velocity. It suggests that this circulation is caused by Ekman pumping with increasing the rotation speed and thinning the velocity boundary layer on the side wall perpendicular to the magnetic field.

Additional 3D flow structures appear as couple of thinner vortices advecting along the horizontal circulation. Appearance of the vortices modulates the behaviour of primary convection rolls with a frequency that is about one order lower as the oscillation frequency of the rolls. The existence of the vortices is related to smaller vortices aligned parallel to the primary convection rolls. These vortices become unstable at a certain Ra/Q threshold and wind around the convection rolls. The range to observe this vortical structure on Ra/Q is very close to that for the flow reversals, $Ra/Q \sim 10$, appearance of the structure may restrict the appearance of the flow reversals.

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