Measurements of a three-dimensional flow in a cube by means of an ultrasound array Doppler velocimeter (UADV)

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Velocity measurements were carried out in a cube filled with the liquid metal GaInSn using a dual plane, two-component ultrasound array Doppler velocimeter. It provides flow instrumentation at frames rate of typ. 11 Hz over long measurement durations (>1h) using an FPGA based signal processing, which enables capturing transient flow phenomena with nondeterministic onsets. The liquid metal was suddenly exposed to an azimuthal body force generated by a rotating magnetic field (RMF). The measurements show a similar flow structure compared to the case of the RMF-driven flow in a cylindrical container, in particular the so-called initial adjustment phase followed by an inertial phase which is dominated by inertial oscillations of the secondary flow. The interest was especially focused on the onset of unsteady flow regimes. The transition from the steady double vortex structure of the secondary flow to an oscillating regime was detected at a magnetic Taylor number of $Ta > 1.3\times10^5$. A detailed analysis of the flow structure was done by means of the Proper Orthogonal Decomposition (POD). Corresponding numerical simulations were performed showing an excellent agreement with the experimental data.

Keywords: Rotating magnetic field driven flow, Ultrasound velocity measurement, Spin-up, OpenFOAM, Proper orthogonal decomposition”

1. Introduction

In the last years a broad study about the flow in closed circular cylindrical container under the action of a rotating magnetic field was performed, paying special attention to the transition from a steady to a time-dependent flow regime [1]. The authors tried to find a critical non-dimensional magnetic Taylor number *Ta* (cf. Eq. 3) for the onset of the instabilities using different direct numerical simulation methods. Most recently, Grants and Gerbeth [2] conclude that the rotating-magnetic field driven flow in a circular cylinder becomes unstable first to non-axisymmetric, azimuthally periodic perturbations at diameter-to-height aspect ratios $AR=D/H$ between 0,5 and 2. In this paper we present an experimental and numerical study of the flow in a closed cubic container driven by a rotating magnetic field. We focused our interest in the transient behavior of the flow and the characterization of the transition from a steady state below the critical magnetic Taylor number ($Ta < Ta_{cr}$) to oscillatory states and then, for a considerable high Ta, to a completely unsteady state.

2. Experimental setup

The experimental setup consists of the eutectic metal alloy GaInSn, which is liquid at room temperature, enclosed in a cubic container with an edge length of $2L$ = 67,5 mm. The container is centered in the MULTIMAG (MULTI purpose MAGnetic field) facility, which is capable of generating different types of magnetic fields with varying strength and frequency [3].

In the current study the investigations were focused on the spin-up process, this means that the experiments were started with the fluid being at rest. The liquid metal (GaInSn) was suddenly exposed to an azimuthal body force generated by a rotating magnetic field (RMF) with a frequency of 50 Hz. Due to the electromagnetic force action and the presence of walls, a vortex develops in the horizontal plane (primary flow) as well as a secondary flow in the meridional plane. Both planes are instrumented simultaneously with the ultrasound array Doppler velocimeter (UADV) [4, 5] as shown in Fig. 1. In the given configuration, the UADV employs four linear arrays.

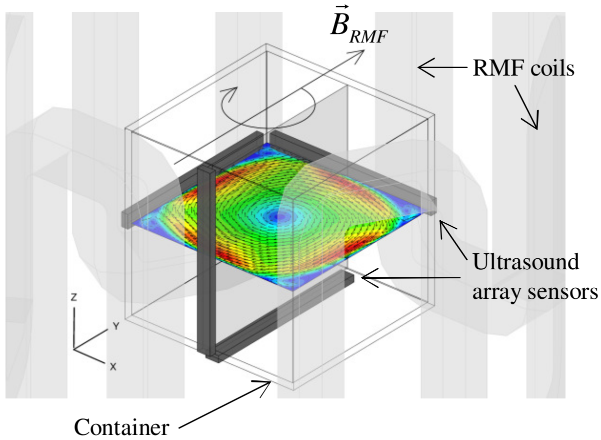

Figure 1: Measurement configuration: cube instrumented with four ultrasound array sensors.

An array consists of 25 transducers with the dimensions 2,5 × 5 mm^2 resulting in a total sensitive length of 67,5 mm (cf. Fig. 2). A pairwise driving of neighboring transducers results in an active surface of 5 × 5 mm^2, which leads to a sound beam width of approximately 3 mm in GaInSn.

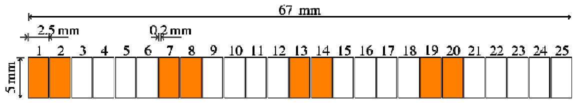

Figure 2: Geometrical dimensions of the used ultrasound array sensor.

By arranging the sensor arrays orthogonally in a single plane and combining the data two components velocity fields can be obtained. The excitation signal is eight periods of a sine wave at 8 MHz resulting in an axial resolution of about 1.4 mm [4]. The acoustical impedance of the transducers is matched to PMMA (3.4 MRayl), which allows reliable measurements through the container walls. The frame-rate is increased over a simple sequential scan by using a parallelized time division multiplexing (TDM) scheme. In this way a measurement frame rate of up to 33 Hz can be achieved. To avoid crosstalk between the sensor arrays, all four arrays are driven mutual exclusively. The velocity information is extracted from the amplified and digitized US echo signals via the Kasai autocorrelation method. A typical mean bandwidth after digitalization is 1.2 GB/s, which is beyond the limit that can be acquired and stored continuously with common PC-hardware. A real-time data compression is performed by offloading parts of the signal processing to a field-programmable gate array (FPGA, NI PXIe-7965R). The pre-processing reduces the amount of data by 10:1 and enables a continuous streaming for a practically unlimited duration. This approach allows for the study of long-term transient flows.

3. Governing equations

Let us consider the flow of an electrically conducting fluid with kinematic viscosity ν, density ρ and electrical conductivity σ in a cubic container with an edge length $2L$ driven by a uniform magnetic field of induction B_0 rotating in a horizontal plane (around the z - axis) with a constant angular frequency ω. In the scope of the low-induction approximation (very small magnetic Reynolds number $Rm = \mu_0 \omega u_0 L << 1$) the electromotive field $\vec{u} \times \vec{B}$ can be neglected compared to the induced electric field E within the Ohm's law $\vec{j} = \sigma(\vec{E} + \vec{u} \times \vec{B})$. Here is μ_0 the magnetic vacuum permeability and u_0 is a characteristic velocity of the flow. The simulations of the electromagnetic field and the fluid flow can be conducted separately. Hence, a quasi-analytical expression for the over one period time-averaged electromagnetic force density $\vec{f} = < \vec{j} \times \vec{B} >_T$ acting on the liquid metal in the cavity can be derived [6]:

$$\vec{f} = \sigma \varpi B_0^2 / 2\{(-\partial_z b + y)\vec{e}_x + (-\partial_z a - x)\vec{e}_y\} \qquad (1)$$

The functions a and b are solutions of the Laplace equation under special boundary conditions [6]. The numerical simulation of the liquid metal flow is performed using the open code library OpenFOAM. The flow was computed solving the incompressible Navier Stokes equation which in a dimensionless form, with L, L^2/ν and $\rho(\nu/L)^2$ being the distance, time and pressure scale, respectively, is given by

$$\partial_t \vec{u} + (\vec{u} \cdot \nabla)\vec{u} = -\nabla p + \nabla^2 \vec{u} + Ta\, \vec{f}_{EM} \qquad (2)$$

together with the incompressibility condition $\nabla \cdot \vec{u} = 0$. The last term is the non-dimensional electromagnetic force density and

$$Ta = \frac{\sigma \varpi B_0^2 L^4}{2\rho \nu^2} \qquad (3)$$

denotes the magnetic Taylor number, a ratio between the electromagnetic and the viscous force acting on the liquid metal.

Boundary conditions for the calculation of the flow field have been the no-slip condition $u = 0$ at the solid container walls. For the upper surface either $u = 0$ or the conditions for a stress-free, non-deformable surface $u_n = 0$ and $\partial u_t / \partial n = 0$ were used depending on whether the melt flow was evaluated in an open or in an enclosed container. A computational grid with at least one million volume elements was used depending on the boundary layer thickness. A second-order discretization scheme was used for the convective term in Eq. 2.

4. Results

In order to estimate the critical Taylor number Ta_{cr} for the transition from steady to oscillatory flow regime, we examine the time evolution of the velocity field. Fig. 3 shows the velocity component u_x at the monitoring point with the coordinates $x = 0{,}9L;\ y = 0;\ z = 0{,}1\ L$ for different values of Ta as a function of time. The transition to a time-dependent flow regime appears to occur for a value of the magnetic Taylor number in the range $1{,}3\times10^5 < Ta_{cr} < 1{,}4\times10^5$. The corresponding critical value for the case of a finite circular cylinder of aspect ratio 1 is $Ta_{cr} = 1{,}232\times10^5$ (Grants et al. [2]).

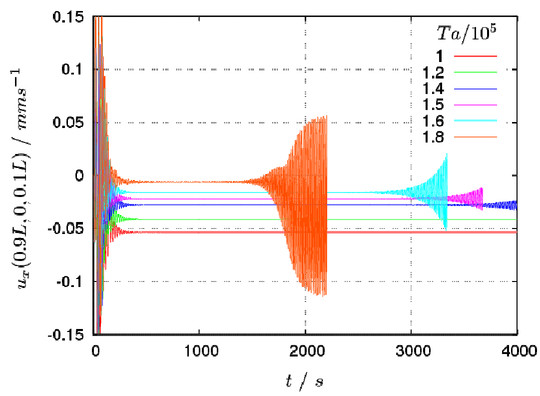

Figure 3: Time evolution of u_x at a monitoring point for different values of the magnetic Taylor number Ta.

The secondary flow reaches the maximal values near the top and the bottom of the container. Due to saturation effects, the measurement system yields no valid velocity measurements near the walls and therefore the measurement volume does not include these regions.

Fig. 4 shows a comparison of the flow pattern in the meridional plane ($x=0$) for $Ta = 10^5$. In the top part, the values from the numerical simulation are shown. We can observe at the bottom part of this figure that the maximal velocity magnitudes are found outside of the measurement volume of the ultrasound instrumentation. This fact shows the importance of the numerical simulation in order to resolve boundary layers correctly in near wall regions. On the other hand side, the experimental approach in combination with proper flow instrumentation can yield data in the high *Ta* regime, which is not accessible through direct numerical simulation (DNS) yet.

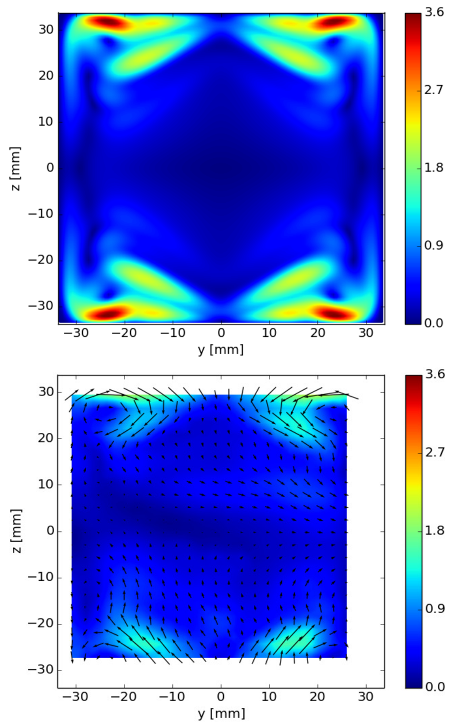

Figure 4: Mean velocity of the secondary flow in mm/s for $Ta = 10^5$ (top: computed from the numerical simulation, bottom: measured mean velocity distribution)

Complex flow time-dependent structures can be described using a model reduction because such flows are often dominated by low-dimensional dynamics. Such a one is the proper orthogonal decomposition (POD) technique, which decomposes the flow velocity vector field into orthogonal spatial modes and time-dependent amplitudes.

Fig. 5 depicts the time evolution of the amplitudes of the leading modes $a_1(t)$, $a_3(t)$, $a_4(t)$ and $a_6(t)$. The upper diagram demonstrates the exponential growth of the modes whereas the evolution of the kinetic energy $a_m(t)^2$ of the modes $m=1,2,\ldots,7$ can be seen in the bottom graph. More details will be shown in a subsequent paper that will be published elsewhere.

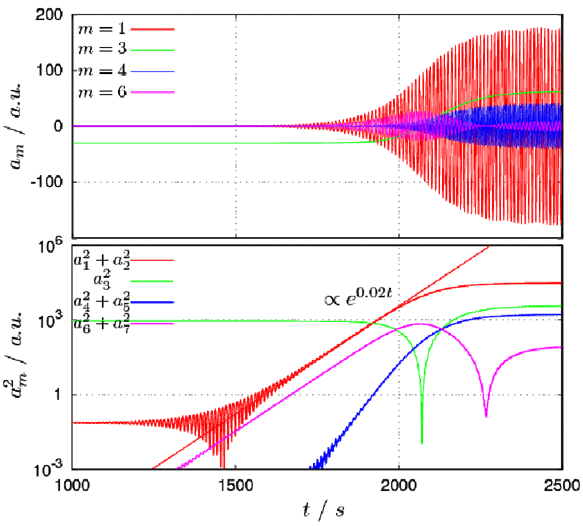

Figure 5: POD of the primary flow (numerical simulation for $Ta = 1{,}7\times10^5$) - Time evolution of the amplitude of the most important modes $a_m(t)$; $m = 1, 3, 4, 6$ (top) and of the kinetic energy of the modes $m = 1,\ldots,7$ (bottom).

4. Conclusions

We present a semi-analytical expression for the induced electromagnetic force density in an electrically conducting medium contained in a square cavity in the presence of a rotating magnetic field. This allows a direct numerical simulation of the flow. We found the transition from the steady state to time dependent flow structures to occur for $Ta>1{,}3\times10^5$ Velocity distributions in two perpendicular planes were measured using a two-component ultrasound array Doppler velocimeter for a large time interval for the first time. The mean flow structures and the evolution of the flow in time in experiments and numerical simulation are in very good agreement.

Acknowledgements

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